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Extensional Vibration Analysis of Curved Beams Including Rotatory Inertia and Shear Deformation Using DQM

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미분구적법(DQM)을 이용 회전관성 및 전단변형을 포함한 곡선 보의 신장 진동해석

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Abstract One of the most efficient procedures for the solution of partial differential equations is the method of differential quadrature. The differential quadrature method (DQM) has been applied to a large number of cases to overcome the difficulties of complex algorithms of computer programming, as well as the excessive use of storage due to the conditions of complex geometry and loading. The in-plane vibrations of curved beams with extensibility of the arch axis, including the effects of rotatory inertial and shear deformation, are analyzed by the DQM. The fundamental frequencies are calculated for members with various slenderness ratios, shearing flexibilities, boundary conditions, and opening angles. The results are compared with the numerical results obtained by other methods for cases in which they are available. The DQM gives good mathematical precision even when only a limited number of grid points is used, and new results according to diverse variations are also suggested.

Keywords : DQM, Extensional Vibration, New Result, Rotatory Inertial, Shear Deformation

요 약 편미분방정식의 해를 구하기 위한 효율적인 방법 중의 하나는 미분구적법이다. 복잡한 기하학적 구조 및 하중 은 컴퓨터 용량을 과도하게 사용할 뿐만 아니라, 복합알고리즘 프로그램을 어렵게 해 이를 극복하기위하여 미분구적법(DQM)이 많은 분야에 적용되어왔다. 곡선 보의 아크 축 신장에 회전관성 및 전단변형을 포함하여 DQM을 이용 곡선 보의 내 평면 진동을 해석하였다. 다양한 세장비 및 전단신축성 그리고 경계조건 및 열림 각에 따른 기본진동수를 계산하였다. DQM 결과는 활용 가능한 다른 수치해석결과와 비교하였다. DQM은 적은 격자점을 사용하고도 정확한 해석을 보여주었고, 다양한 변경에 따른 새로운 결과 또한 제시하였다.

1. Introduction

The increasing use of curved beams in buildings, vehicles, ships, and aircraft has results in considerable effort being directed toward developing an accurate method for analyzing the dynamic behavior of such structures. Accurate knowledge of the vibration response of curved beams is of great importance in many engineering applications such as the design of machines and structures.

The earlier investigators into the in-plane vibration of rings were Hoppe[1] and Love[2]. Love[2] improved on Hoppe's theory by allowing for stretching of the ring. Lamb[3] investigated the statics of

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incomplete ring with various boundary conditions and the dynamics of an incomplete free-free ring of small curvature. Den Hartog[4] used the Rayleigh-Ritz method for finding the lowest natural frequency of circular arcs with simply supported or clamped ends, and his work was extended by Volterra and Morell[5] for the vibrations of arches having center lines in the form of cycloids, catenaries, or parabolas. Archer[6] carried out for a mathematical study of the in-plane inextensional vibrations of an incomplete circular ring of small cross section with the basic equations of motion as given in Love[2] and gave a prescribed time-dependent displacement at the other end for the case of clamped ends. Veletsos et al.[7] used a theory which accurately considered the extensibility of the arch axis and the curved beam effect but neglects the effects of rotatory inertia and shearing deformation. The elementary Bernoulli-Euler equation of motion of beams is derived on the assumption that the deflections of beams are due to bending only and that both transverse shear and rotary inertia are neglected. In addition, it is assumed that the center line remains unextended during bending, while the bending characteristics are ignored when considering the extensional behavior. This is recognized as adequate for the usual engineering problems. However, for beams having large cross-sectional dimensions in comparison to their lengths, and for beams in which high-frequency modes of vibration are required, the Timoshenko theory[8] which takes into account the rotary inertia and shear effects gives a better approximation to the actual beam behavior. The effects of shear deformation and rotary inertia on inextensional vibrations of a circular ring was considered by Roa and Sundararajan[9]. Recently, Issa et al.[10] presented extensional vibrations of continuous circular curved beams with rotary inertia and shear deformation, and Austin and Veletsos[11] presented free vibrations of circular arches flexible in shear, respectively. More recently, Kang and Kim[12] analyzed the in-plane extensional vibration of curved beams using DQM neglecting the effects of shear deformation, Kang and Kim[13] studied the out-of-plane vibration with shear deformation but not considering the extensibility of the arch axis, and Kang[14] analyzed the inextensional and extensional vibrations of curved beams neglecting the effects of rotatory inertial and shear deformation.

A rather efficient alternate procedure for the solution of partial differential equations is the method of differential quadrature which was introduced by Bellman and Casti[15]. This simple direct technique can be applied to a large number of cases to circumvent the difficulties of programming complex algorithms for the computer, as well as excessive use of storage. This method is used in the present work to analyze the free in-plane extensional vibrations of curved beams with various slenderness ratio, shearing flexibility, boundary conditions, and opening angles including the effects of rotatory inertial and shear deformation. The results are compared with numerical solutions by other methods.

2. Differential Equations

The uniform curved beam considered is shown in Fig. 1. A point on the centroidal axis is defined by the angle θ, measured from the left support. The tangential and radial displacements of the arch axis are v and w, respectively. Here a is the radius of the centroidal axis, and θ_0 is the opening angle.

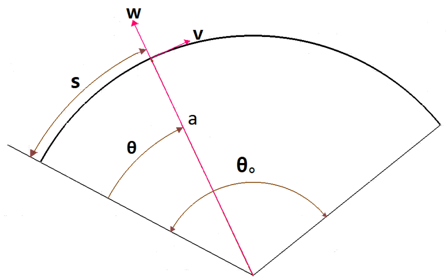

Fig. 1. Coordinate system for curved beam

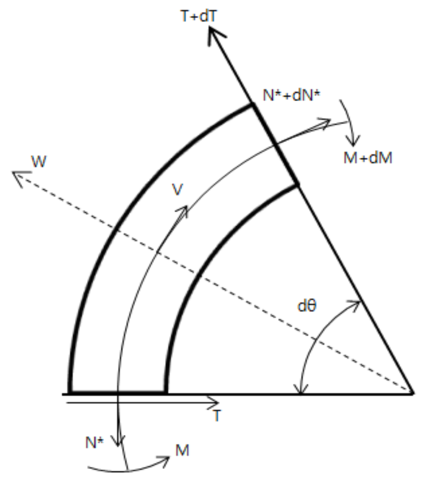

Fig. 2. Curved element subjected to forces

The equilibrium conditions of a circular cured beam element, undergoing small undamped in-plane vibration as shown in Fig. 2, give[9]

$$\frac{\partial T}{\partial \theta} + N^* = ma\frac{\partial^2 w}{\partial t^2} \tag{1}$$

$$\frac{\partial N^*}{\partial \theta} - T = ma\frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$-\frac{\partial M}{\partial \theta} + Ta = \frac{maI}{A}\frac{\partial^2 \psi}{\partial t^2} \tag{3}$$

Where N^*, T, and M are the normal force, internal shear force, and bending moment, respectively. Here ψ is the bending slope, m is the mass per unit length, A is the cross sectional area, t is the time, and I is the area moment of inertia of the cross section. The total angle ϕ between the deformed and undeformed neutral axis may be expressed as[8]

$$\phi = \psi + \beta \tag{4}$$

Where β is the angular deformation due to shear.

From the elementary theory of beams, the normal force, the bending moment, and shear force are given

$$N^* = \left(\frac{EA}{a}\right)\left(\frac{\partial v}{\partial \theta} - w\right) \tag{5}$$

$$M = -\left(\frac{EI}{a}\right)\frac{\partial \psi}{\partial \theta} \tag{6}$$

$$T = \left(\frac{kAG}{a}\right)\left(\frac{\partial w}{\partial \theta} + v - a\psi\right) \tag{7}$$

where E is the Young's modulus of elasticity, k is the numerical shape factor of cross-section, and G is the shear modulus.

Assume that the beam is undergoing free vibration with a frequency ω and let

$$v(\theta,t) = V(\theta)e^{i\omega t},\ w(\theta,t) = W(\theta)e^{i\omega t},$$
$$\psi(\theta,t) = \Psi(\theta)e^{i\omega t} \tag{8}$$

where $i = \sqrt{-1}$, $V(\theta)$ is the normal function of $v(\theta)$, $W(\theta)$ is the normal function of $w(\theta)$, and $\Psi(\theta)$ is the normal function of $\psi(\theta)$, respectively.

Substituting equations (5), (6), and (7) with equation (8) into equations (1), (2), and (3) and omitting the common term $e^{i\omega t}$ yields

$$-\left(\frac{Aa^2}{I}\right)W'' + \left(\frac{kGAa^2}{EI}\right)W + \left(\frac{Aa^2}{I} + \frac{kGAa^2}{EI}\right)U'$$
$$-\left(\frac{kGAa^3}{EI}\right)\Psi = \frac{ma^4\omega^2}{EI}W \tag{9}$$

$$-\left(\frac{Aa^2}{I} + \frac{kGAa^2}{EI}\right)W' - \left(\frac{kGAa^2}{EI}\right)U'' + \left(\frac{Aa^2}{I}\right)U$$
$$+\left(\frac{kGAa^3}{EI}\right)\Psi' = \frac{ma^4\omega^2}{EI}U \tag{10}$$

$$-\left(\frac{Aa^2}{I}\right)\left(\frac{kGAa^2}{EI}\right)W - \left(\frac{Aa^2}{I}\right)\left(\frac{kGAa^2}{EI}\right)U' - \left(\frac{Aa^3}{I}\right)\Psi''$$
$$+\left(\frac{Aa^2}{I}\right)\left(\frac{kGAa^3}{EI}\right)\Psi = \frac{ma^5\omega^2}{EI}\Psi \tag{11}$$

in which each prime denotes one differentiation with respect to the angular coordinate θ.

Using the length of the arch axis S, the radius of gyration of the cross section r, the non-dimensional frequency parameter λ, and the dimensionless distance coordinate Y, the equations (9), (10), and (11) can be written as

$$-(\frac{S}{\theta_0 r})^2 \frac{W''}{\theta_0^2} + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 W + ((\frac{S}{\theta_0 r})^2 + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2)\frac{U'}{\theta_0} - (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a\Psi = \lambda^2 W \quad (12)$$

$$-((\frac{S}{\theta_0 r})^2 + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2)\frac{W'}{\theta_0} - (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 \frac{U''}{\theta_0^2} + (\frac{S}{\theta_0 r})^2 U + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a\frac{\Psi'}{\theta_0} = \lambda^2 U \quad (13)$$

$$-(\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 W - (\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 \frac{U'}{\theta_0}$$

$$-(\frac{S}{\theta_0 r})^2 a\frac{\Psi''}{\theta_0^2} + (\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a\Psi = \lambda^2 a\Psi \quad (14)$$

where S, r, and λ are given by, respectively,

$$S = a\theta_0, \quad r = \sqrt{\frac{I}{A}}, \quad \lambda = \sqrt{\frac{ma^4\omega^2}{EI}} \quad (15)$$

in which each prime denotes one differentiation with respect to the dimensionless distance coordinate Y defined as

$$Y = \frac{\Theta}{\Theta_0} \quad (16)$$

Neglecting the effects of rotatory inertia and shear deformation, the differential equations can be written as[12]

$$\frac{W''''}{\theta_0^4} + 2\frac{W''}{\theta_0^2} + [1 + (\frac{S}{\theta_0 r})^2] W + (\frac{S}{\theta_0 r})^2 \frac{V'}{\theta_0} = \lambda^2 W \quad (17)$$

$$-(\frac{S}{r\theta_0})^2 [\frac{W'}{\theta_0} + \frac{V''}{\theta_0^2}] = \lambda^2 V \quad (18)$$

Neglecting the effects of shear deformation but including rotatory inertia, the differential equations can be written as[12]

$$\frac{W''''}{\theta_0^4} + 2\frac{W''}{\theta_0^2} + [1 + (\frac{S}{\theta_0 r})^2] W + (\frac{S}{\theta_0 r})^2 \frac{V'}{\theta_0} = \lambda^2 (-(\frac{r}{S})^2 W'' + W + (\frac{r}{S})^2 \theta_0 V') \quad (19)$$

$$\frac{W'}{\theta_0^3} + \frac{V''}{\theta_0^4} = \lambda^2 (\theta_0 (\frac{r}{S})^4 W' - ((\frac{r}{S})^2 + (\theta_0^2 (\frac{r}{S})^4)) V) \quad (20)$$

The boundary conditions for clamped and simply supported ends are, respectively.

$$W(0) = V(0) = \Psi(0) = W(\theta_0) = V(\theta_0) = \Psi(\theta_0) = 0 \quad (21)$$

$$W(0) = V(0) = \Psi'(0) = W(\theta_0) = V(\theta_0) = \Psi'(\theta_0) = 0 \quad (22)$$

3. Differential Quadrature Method

The differential quadrature method (DQM) was introduced by Bellman and Casti[15]. By formulating the quadrature rule for a derivative as an analogous extension of quadrature for integrals in their introductory paper, they proposed the differential quadrature method as a new technique for the numerical solution of initial value problems of ordinary and partial differential equations. It was applied for the first time to static analysis of structural components by Jang et al.[16]. The versatility of the DQM to engineering analysis in general and to structural analysis in particular is becoming increasingly evident by the related publications of recent years. Kang and Kim[17] studied the in-plane buckling analysis of curved beams using DQM. Recently, Kang and Kim[18], and Kang and Park[19] studied the vibration and the buckling analysis of asymmetric curved beams using DQM, respectively. From a mathematical point of view, the application of the differential quadrature method to a partial differential equation can be expressed as follows:

$$L\{f(x)\}_i = \sum_{j=1}^{N} W_{ij} f(x_j) \quad \text{for } i, j = 1,1,3,..,N \tag{23}$$

where L denotes a differential operator, x_j are the discrete points considered in the domain, i are the row vectors of the N values, $f(x_j)$ are the function values at these points, W_{ij} are the weighting coefficients attached to these function values, and N denotes the number of discrete points in the domain. This equation, thus, can be expressed as the derivatives of a function at a discrete point in terms of the function values at all discrete points in the variable domain.

The general form of the function $f(x)$ is taken as

$$f_k(X) = X^{k-1} \quad \text{for } k = 1,2,3,...,N \tag{24}$$

If the differential operator L represents an n^{th} derivative, then

$$\sum_{j=1}^{N} W_{ij} x_j^{k-1} = (k-1)(k-2)\cdots(k-n) x_i^{k-n-1} \quad \text{for } i, k = 1,2,\ldots,N \tag{25}$$

This expression represents N sets of N linear algebraic equations, giving a unique solution for the weighting coefficients, W_{ij}, since the coefficient matrix is a Vandermonde matrix which always has an inverse.

4. Numerical Application

The DQM is applied to the determination of the in-plane extensional vibrations of the curved beam including the effects of rotatory inertia and shear deformation. The differential quadrature approximations of the governing equations and boundary conditions are shown.

Applying the differential quadrature method to equations (12), (13), and (14) gives

$$-(\frac{S}{\theta_0 r})^2 \sum_{j=1}^{N} B_{ij} \frac{W_j}{\theta_0^2} + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 W_i + ((\frac{S}{\theta_0 r})^2 + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2) \sum_{j=1}^{N} A_{ij} \frac{U_j}{\theta_0} - (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a\Psi_i = \lambda^2 W_i \tag{26}$$

$$-((\frac{S}{\theta_0 r})^2 + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2) \sum_{j=1}^{N} A_{ij} \frac{W_j}{\theta_0} - (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 \sum_{j=1}^{N} B_{ij} \frac{U_j}{\theta_0^2} + (\frac{S}{\theta_0 r})^2 U_i + (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a \sum_{j=1}^{N} A_{ij} \frac{\Psi_j}{\theta_0} = \lambda^2 U_i \tag{27}$$

$$-(\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 W_i - (\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 \sum_{j=1}^{N} A_{ij} \frac{U_j}{\theta_0} - (\frac{S}{\theta_0 r})^2 a \sum_{j=1}^{N} B_{ij} \frac{\Psi_j}{\theta_0^2} + (\frac{S}{\theta_0 r})^2 (\frac{kG}{E})(\frac{S}{\theta_0 r})^2 a\Psi_i = \lambda^2 a\Psi_i \tag{28}$$

The boundary conditions for clamped ends, given by equation (21), can be expressed in differential quadrature form as follows:

$$W_1 = 0 \quad \text{at} \quad Y = 0 \tag{29}$$
$$W_N = 0 \quad \text{at} \quad Y = 1 \tag{30}$$
$$V_1 = 0 \quad \text{at} \quad Y = 0 \tag{31}$$
$$V_N = 0 \quad \text{at} \quad Y = 1 \tag{32}$$
$$\Psi_1 = 0 \quad \text{at} \quad Y = 0 \tag{33}$$
$$\Psi_N = 0 \quad \text{at} \quad Y = 1 \tag{34}$$

Similarly, the boundary conditions for simply supported ends, given by equation (22), can be expressed in differential quadrature form as follows:

$$W_1 = 0 \quad \text{at} \quad Y = 0 \tag{35}$$
$$W_N = 0 \quad \text{at} \quad Y = 1 \tag{36}$$
$$V_1 = 0 \quad \text{at} \quad Y = 0 \tag{37}$$
$$V_N = 0 \quad \text{at} \quad Y = 1 \tag{38}$$

$$\sum_{j=1}^{N} A_{1j}\Psi_j = 0 \qquad \text{at} \quad Y=0 \tag{39}$$

$$\sum_{j=1}^{N} A_{Nj}\Psi_j = 0 \qquad \text{at} \quad Y=1 \tag{40}$$

This set of equations together with the appropriate boundary conditions can be solved for the in-plane extensional vibrations of the curve beam including the effect of rotatory inertia and shear deformation.

5. Numerical Results and Comparisons

Based on the above derivations, the fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$ or $\lambda^* = \omega S^2(m/EI)^{1/2}$, of the in-plane extensional vibrations of the curved beam including the effects of rotatory inertial and shear deformation are calculated by the DQM and are presented together with numerical solutions by other methods. Fig. 3 presents the results of convergence studies relative to the number of grid points N. Fig. 3 shows that the accuracy of the numerical solution increases with increasing N and passes through a maximum. Then, numerical instabilities arise if N becomes too large. The optimal value for N is found to be 11 to 15. All results are computed with thirteen discrete points along the dimensionless axis.

The fundamental frequency parameter $\lambda^* = \omega S^2(m/EI)^{1/2}$ including the effects of rotatory inertial and shear deformation is calculated by the DQM for the comparisons and is presented together with the results by Austin and Veletsos[11] using a combination of a Holzer-type iterative procedure. The results are summarized in Tables 1~2, and the solutions by the DQM give the good accuracy compared with the solutions by Austin and Veletsos[11].

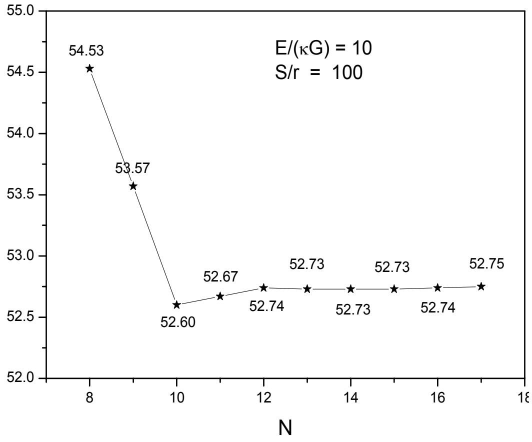

Fig. 3. Fundamental frequency parameters, $\lambda^* = \omega S^2(m/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertia and shear deformation with clamped-clamped ends including a range of N; $\theta_0 = 90^0$, E/kG = 10, and S/r=100

In Tables 3~8, the fundamental frequency parameter $\lambda = \omega(ma^4/EI)^{1/2}$ of the in-plane extensional vibrations of the beam including the effects of rotatory inertial and shear deformation is calculated by the DQM. Tables 3~4, Tables 5~6, and Tables 7~8 show the fundamental frequency parameter $\lambda = \omega(ma^4/EI)^{1/2}$ with clamped-clamped, simply-simply supported, and clamped-simply supported ends with variation of the shearing flexibility E/kG and the slenderness ratio S/r, respectively. There are no comparisons since no data are available.

Table 1. Fundamental frequency parameters, $\lambda^* = \omega S^2(m/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with clamped-clamped ends; E/kG = 10/3 and $\theta_0 = 90^0$

S/r	$\lambda^* = \omega S^2(m/EI)^{1/2}$	
	Austin and Veletsos[11]	DQM
25	42.44	42.26
50	51.38	51.53
75	53.72	53.69
100	54.63	54.60
150	55.28	55.28
200	55.53	55.52
300	55.69	55.70

Table 2. Fundamental frequency parameters, $\lambda^* = \omega S^2(m/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with clamped-clamped ends; E/kG = 10 and $\theta_0 = 90^0$

S/r	$\lambda^* = \omega S^2(m/EI)^{1/2}$	
	Austin and Veletsos[11]	DQM
25	32.60	32.50
53	46.72	46.65
100	52.76	52.73
200	55.03	55.01
300	55.47	55.46
400	55.63	55.63

Table 3. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with clamped-clamped ends; E/kG = 3

θ_0, degrees	S/r			
	30	50	100	300
30	88.61	111.4	175.3	221.9
60	29.80	43.15	55.71	53.63
90	17.17	20.93	22.17	22.57
120	9.689	10.92	11.59	11.82
150	5.667	6.404	6.808	6.943
180	3.570	4.033	4.289	4.374

Table 4. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with clamped-clamped ends; E/kG = 5

θ_0, degrees	S/r			
	30	50	100	300
30	86.07	110.5	175.1	221.7
60	29.39	42.96	52.16	53.57
90	16.99	20.17	21.93	22.55
120	8.925	10.52	11.47	11.80
150	5.233	6.164	6.733	6.934
180	3.294	3.881	4.241	4.368

Table 5. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with simply-simply supported ends; E/kG = 3

θ_0, degrees	S/r			
	30	50	100	300
30	62.00	92.37	140.42	141.4
60	26.72	32.54	33.34	33.60
90	12.56	13.29	13.64	13.75
120	6.302	6.681	6.863	6.920
150	3.504	3.719	3.822	3.855
180	2.058	2.184	2.245	2.264

Table 6. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with simply-simply supported ends; E/kG = 5

θ_0, degrees	S/r			
	30	50	100	300
30	61.79	92.29	139.8	141.3
60	26.65	32.07	33.31	33.58
90	12.14	13.10	13.59	13.74
120	6.088	6.585	6.836	6.917
150	3.385	3.665	3.807	3.853
180	1.988	2.153	2.236	2.263

Table 7. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curve beams including the effects of rotatory inertial and shear deformation with clamped-simply supported ends; E/kG = 3

θ_0, degrees	S/r			
	30	50	100	300
30	72.27	98.11	165.5	178.6
60	27.31	39.37	42.29	42.88
90	15.07	16.84	17.60	17.84
120	7.859	8.697	9.070	9.196
150	4.546	4.992	5.2177	5.291
180	2.794	3.064	3.204	3.248

Table 8. Fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for in-plane extensional vibrations of curved beams including the effects of rotatory inertial and shear deformation with clamped-simply supported ends; E/kG = 5

θ_0, degrees	S/r			
	30	50	100	300
30	71.36	97.84	165.0	178.4
60	27.13	38.63	41.99	42.84
90	14.36	16.41	17.48	17.83
120	7.462	8.455	9.004	9.188
150	4.289	4.861	5.179	5.286
180	2.634	2.984	3.180	3.246

From Tables 3 ~ 8, the frequencies of the member with clamped ends are much higher than those of the member with simply supported ends and clamped-simply supported ends. The frequencies can be increased by decreasing the opening angle θ_0 and the shearing flexibility E/kG . The frequencies can be also increased by increasing the slenderness ratio S/r in the cases of all boundary conditions.

In Figs. 4~7, the fundamental frequency parameters of the in-plane extensional vibrations of the beam neglecting or including the effects of rotatory inertial and shear deformation with both ends clamped (C-C), simply supported (S-S), and clamped-simply supported ends (C-S) are compared with the cases of the slenderness ratio S/r are 30 and 300, the shearing flexibility E/kG are 3 and 10, and the opening angle are 180 degree.

From Figs. 4 ~ 7, the frequency parameters of the vibrations neglecting the effects of rotatory inertial and shear deformation using equations (17)~(18) are higher than those of the vibrations including the effects of rotatory inertial only using equations (19)~(20), and the frequency parameters of the vibrations including the effects of rotatory inertial only are also higher than those of the vibrations including both the effects of rotatory inertial and shear deformation using equations (12)~(14). The difference of the fundamental frequency values at the same opening θ_0 is decreased by increasing the slenderness ratio S/r and decreasing the shearing flexibility E/kG. When the slenderness ratio is greater than 300, the frequency values are less affected by rotatory inertial and shear deformation, and the values become almost the same (less than 0.5 percent). The frequencies of the member with clamped- clamped ends are more affected by the slenderness ratio S/r and the shearing flexibility E/kG than those with any other boundary conditions.

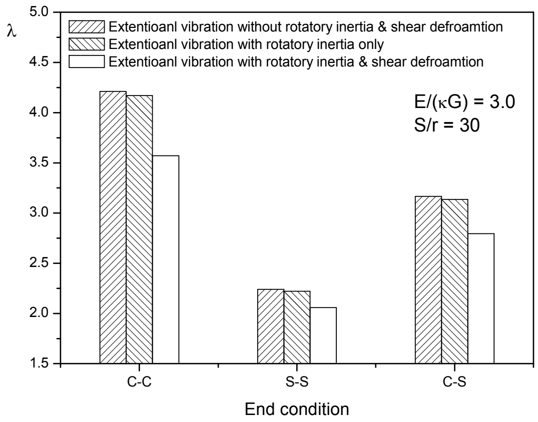

Fig. 4. Comparisons between fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for extensional vibrations of the curved beam with E/kG=3, S/r = 30, and θ_0 = 180^0

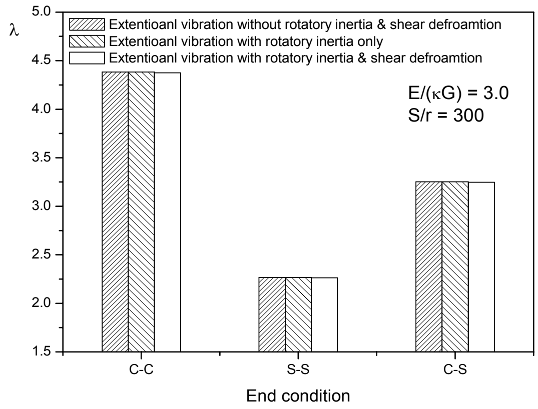

Fig. 5. Comparisons between fundament frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for extensional vibrations of the curved beam with E/kG=3, S/r = 300, and θ_0 = 180^0

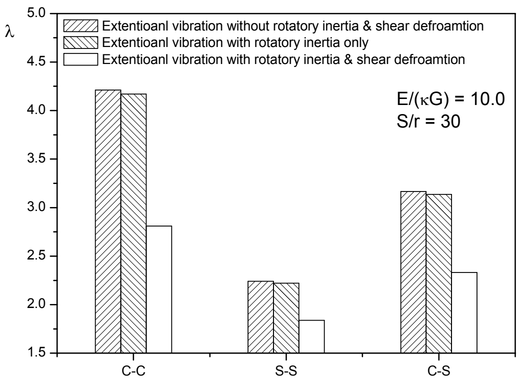

Fig. 6. Comparisons between fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for extensional vibrations of the curved beam with E/kG=10, S/r = 30, and $\theta_0 = 180^0$

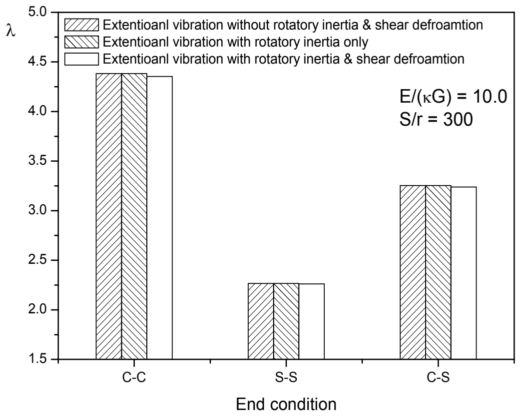

Fig. 7. Comparisons between fundamental frequency parameters, $\lambda = \omega(ma^4/EI)^{1/2}$, for extensional vibrations of the curved beam with E/kG=10, S/r = 300, and $\theta_0 = 180^0$

As it can be seen, the frequency parameters of the beam including the effects of rotatory inertial and shear deformation affect the beam behavior significantly. Therefore, the vibration analysis of curved beams with rotatory inertial and shear deformation is necessary.

6. Conclusions

The differential quadrature method (DQM) was used to compute the eigenvalues of the equations of motion governing the free in-plane extensional vibrations of a curved beam including the effects of rotatory inertial and shear deformation with various the slenderness ratio, shearing flexibility, boundary conditions, and opening angles. The results are analyzed, compared with numerical solutions by other methods (Holzer-type iterative procedure and an initial value integration procedure), and are also compared with each other for the cases of both neglecting the effects of rotatory inertial and shear deformation or including the effects of rotatory inertial and shear deformation. The present method gives the results which agree very well with other solutions for the cases treated while requiring only a limited number of grid points.

The present approach gives the followings:

1) The results by the DQM give the mathematical precision compared with the numerical solutions by others for the cases in which they are available.
2) Only thirteen discrete points are used for the evaluation.
3) It requires less than 1.0 second to compile the program with IMSL subroutine using a personal computer.
4) Diversity of new results according to the opening angles, boundary conditions, shearing flexibility, and slenderness ratio is also suggested. Those results can be used in the comparisons with other numerical solutions or experimental test data.

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