Journal of the Korea Academia-Industrial cooperation Society
Vol. 19, No. 12 pp. 32-40, 2018

https://doi.org/10.5762/KAIS.2018.19.12.32
ISSN 1975-4701 / eISSN 2288-4688

Eigen-Frequency of a Cantilever Beam Restrained with Added Mass and Spring at Free End or a Node Point

Woo-Gun Sim
Department of Mechanical Engineering, Hannam University

자유단 혹은 노드점에 작용하는 스프링과 부가질량을 받는 일단 지지보의 고유진동수

심우건
한남대학교 기계공학과

Abstract In order to avoid excessive vibration, it is required to carry out a vibration analysis of heat-exchanger/ nuclear- reactor at the design stage. Information of eigen-frequency in the vibration problem is required to evaluate safety of heat-exchange/nuclear reactor. This paper describes a numerical method, Galerkin's method, to solve the eigenvalue problem occurred in a cantilever beam. The beam is restrained with added mass and spring at the free end or a node point of a mode shape. The numerical results of eigen-frequency were compared with simple analytical and experimental results given by simple approach and simple test, respectively. It is found that Galerkin's method is applicable to estimate the eigen-frequency of the cantilever beam. The frequencies become lower with increasing the added mass and the frequencies increase with the spring force. It is shown the heavy added mass has a role of support on the flexible tube. The eigen-frequency of the first mode, for the system with the added mass mounted at the free end, can be calculated by the approximate analytical method existing with more or less accuracy.

요 약 열 교환기/원자로의 과도한 진동을 방지 하려면 진동해석을 설계 단계에서 수행해야 한다. 진동 문제에서 고유 진동수의 정보는 열 교환기/원자로의 안전성을 평가하기 위하여 요구된다. 본 논문은 일단 지지보에 발생되는 고유치 문제를 해석하기 위하여 수치해석 방법인 Galerkin의 방법을 기술하였다. 일단 지지보는 자유단 끝점 또는 모드의 노드 포인트에 부가 질량과 스프링에 의하여 구속되어 있다. 수치해석으로 구한 고유진동수는 간단한 해석 방법과 간단한 테스트에 의하여 각각 구한 결과와 비교 되었다. Galerkin의 방법을 사용하여 논의된 일단 지지보의 고유 진동수를 구할 수 있음을 보였다. 부가 질량 증가함에 따라 고유 주파수는 감소하며 스프링 힘의 증가에 따라 고유 주파수는 상승함을 보였다. 무거운 부가 질량은 가연성 배관의 지지대 역할을 함을 보였다. 일단 지지보의 끝단에 설치된 부가 질량의 경우에 개발된 기존의 어림적 해석 방법으로도 일차 모두의 고유 진동수를 비교적 정확하게 구할 수 있음을 알 수 있었다.

Keywords : Added Mass, Cantilever Beam, Eigen-frequency, Flow-induced-Vibration, Galerkin's Method

1. Introduction

Tubular heat exchanger in steam generator of nuclear power plant is representative structure, subjected to cross flow. Flow-induced vibration of steam generator heat pipe is caused by cross flow [1-3]. Between the tube-to-tube and the tube support, mechanical wear is occurred by the flow-induced

This paper was supported by the 2017 Hannam University Research Fund and I would like to thank for the support.
*Corresponding Author : Woo-Gun Sim(Hannam Univ.)
Tel: +82-10-7184-8089 email: wgsim@hnu.kr
Received September 17, 2018 Revised (1st November 5, 2018, 2nd November 14, 2018)
Accepted December 7, 2018 Published December 31, 2018

vibration. Excessive vibration arising from fluid-elastic instability often leads to fretting wear damages or tube failures in the heat exchangers [4]. Such tube failures can be avoided by a comprehensive vibration analysis at the design stage. The wear and tear, affected on the thickness of the tube, is acting as the main causes that shorten the life of the equipment. The integrity of steam-generator tubes is an important aspect of the long term reliable operation of nuclear power plants.

Design guidelines were recently developed to predict tube failures given by excessive flow-induced vibration in shell-and tube heat exchanger (Pettigrew *et al;* [5]). The Fluid-elastic instability coefficient for tube bundles, defined by Connor [6], was calculated and then compared to existing results given by Sim and Park [7]. From a practical design point of view, critical flow velocity due to fluid-elastic instability may be expressed simply in terms of frequency, fluid-elastic instability coefficient and dimensionless mass-damping parameter [6], as follows;

$$\frac{U_{pc}}{fd} = K\left(\frac{2\pi\zeta m}{\rho d^2}\right)^n . \qquad (1)$$

The eigen-frequency obtained by the present method is used to evaluate the frequency (f) of the tube, shown in the above equation.

To study vibration behavior of tube bundles subjected to two-phase cross-flow, experiments on four tube bundle configurations were conducted by Pettigrew *et al;* [8]. Flow-induced vibration tests were recently conducted on a parallel triangular tube array subjected to Freon two-phase flow (Feenstra et al; [9]). In order to focus future studies and to revise design guidelines, it is required to collect data of existing test results for damping and fluid-elastic instability. Also, it is required to carry out a vibration analysis of heat-exchanger/nuclear-reactor at the design stage. Dynamics and stability of a flexible cylinder in a narrow coaxial cylindrical duct subjected to annular flow was investigated by Paidoussis *et al;* [10].

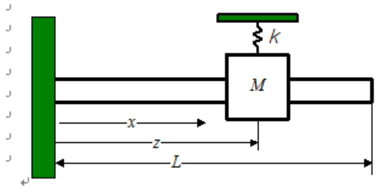


Fig. 1. Schematic diagram of the clamped-free beam with the added mass at $x = z(=\zeta L)$.

Flow-induced vibration excited in cantilever beam may cause excessive vibration. Such a cantilever beam restrained with added mass and spring is used in steam generator or fuel bundle of nuclear steam supply system. To avoid the vibration on the beam, it is required to understand the eigenvalue problems of a cantilever beam. Galerkin's method is one of numerical methods to solve the eigenvalue problem. An availability of the Galerkin's method is discussed with exact solution of a cantilever beam. The beam is restrained with added mass at the free end or a node point of a mode shape. A simple experiment was undertaken for the verification of the Galerkin's method. The effects of the added mass and the spring on the eigen-frequency is evaluated. The Galerkin's method is helpful to calculate the eigen-frequency for the vibration problem of the cantilever beam.

2. Problem Formulation

2.1 Fundamental Theory

As shown in Fig 1. a cantilever beam is restrained with added mass and spring at the free end or a node point of a mode shape. It is required to obtain the eigen-frequency analytically or experimentally, to investigate the effect of the added mass and the spring on the circular frequency. Equation (2) is an Euler momentum equation of a clamped-free beam, shown in Fig.1, with Dirac-delta function at the restrained point. This means an added mass, M, is imposed at $x=z$ of the cantilever beam.

$$EI\frac{\partial^4 y}{\partial x^4}+k\delta(x-z)y+[m+M\delta(x-z)]\frac{\partial^2 y}{\partial t^2}=0. \quad (2)$$

where m is the mass of tube per unit length.

2.2 Eigenvalue Problem

Exact solution without added mass at the free end is expressed as

$$y(t,x)|=\sum_i^n \phi_i(x)q_i(t) \quad (3)$$

where function of mode shape, $\phi(x)$, has sinusoidal and exponential components;

$$\phi_k(x)=-\cos(\beta_k x/L)+\sigma_k\sin(\beta_k x/L)+\cosh(\beta_k x/L)-\sigma_k\sinh(\beta_k x/L)=\phi_k(\xi). \quad (4)$$

To solve the problem, following derivatives with respect to x are needed to define as,

$$\phi_k'(x)=\frac{\beta_k}{L}\begin{pmatrix}\sin(\beta_k x/L)+\sigma_k\cos(\beta_k x/L)\\+\sinh(\beta_k x/L)-\sigma_k\cosh(\beta_k x/L)\end{pmatrix}=\frac{\beta_k}{L}\phi_{1k}(\xi),$$

$$\phi_k''(x)=\frac{\beta_k^2}{L^2}\begin{pmatrix}\cos(\beta_k x/L)-\sigma_k\sin(\beta_k x/L)\\+\cosh(\beta_k x/L)-\sigma_k\sinh(\beta_k x/L)\end{pmatrix}=\frac{\beta_k^2}{L^2}\phi_{2k}(\xi) \quad (5)$$

$$\phi_k''''(x)=\frac{\beta_k^4}{L^4}\begin{pmatrix}-\cos(\beta_k x/L)+\sigma_k\sin(\beta_k x/L)\\+\cosh(\beta_k x/L)-\sigma_k\sinh(\beta_k x/L)\end{pmatrix}=\frac{\beta_k^4}{L^4}\phi_{4k}(\xi)$$

where

$$\sigma_k=(\cos\beta_k+\cosh\beta_k)/(\sin\beta_k+\sinh\beta_k),\quad \xi=x/L \quad (6)$$

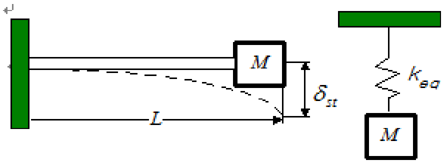
and circular frequency for each mode is expressed in term of eigen-frequency.

$$\omega_k=\beta_k^2\left(\frac{EI}{mL^4}\right)^{0.5} \quad (7)$$

where $\beta_k=1.875,\ 4.694,\ 7.855..$

2.2 Galerkin's Method

Galerkin's Method is used to obtain the eigen-frequency for the system with mass M and stiffness k, mounted at $x=z(=\zeta L)$. Applying the exact solution of eq (4) to eq. (2), following equation can be obtained,



(a) vibratory system, (b) equivalent system

Fig. 2. Equivalent spring.

$$\sum_i\left(EI\frac{\partial^4\phi_i}{\partial x^4}+k\delta(x-z)\phi_i+\right)q_i(t)-\sum_j\omega^2[m+m_a+M\delta(x-z)]\phi_i q_i(t)=\varepsilon \quad (8)$$

Based on Galerkin's method, the equation of motion defined in Eq. (2) is rewritten as

$$\int_o^L\varepsilon\phi_j(x)dx\Big|=0,\qquad \Lambda=\omega^2, \quad (9)$$

and the equation is defined in matrix form:

$$\sum_i^n(k_{ij}-\omega^2 m_{ij})q_i(t)=0 \quad (10)$$

In the above equation, stiffness term is expressed as

$$k_{ij}=\frac{EI\beta_i^4}{L^3}\int_0^1\phi_i(\xi)\phi_j(\xi)d\xi+kL\,\delta(\xi-\zeta)\int_0^1\phi_i(\xi)\phi_j(\xi)d\xi=$$

$$\begin{bmatrix}\frac{EI}{L^3}\beta_1^4+kL\Phi_{11} & kL\Phi_{12} & kL\Phi_{13} & . & kL\Phi_{1n}\\ kL\Phi_{21} & \frac{EI}{L^3}\beta_2^4+kL\Phi_{22} & kL\Phi_{23} & . & kL\Phi_{2n}\\ kL\Phi_{31} & kL\Phi_{32} & \frac{EI}{L^3}\beta_3^4+kL\Phi_{33} & . & kL\Phi_{3n}\\ . & . & . & . & .\\ kL\Phi_{n1} & kL\Phi_{n2} & kL\Phi_{n3} & . & \frac{EI}{L^3}\beta_n^4+kL\Phi_{nn}\end{bmatrix} \quad (11)$$

and mass term

$$m_{ij}=[mL+M\delta(\xi-\zeta)]\int_0^1\phi_i(\xi)\phi_j(\xi)d\xi$$

$$=M\left[\frac{1}{a}\delta_{ij}+\phi_i(\zeta)\phi_j(\zeta)\right]$$

$$=M\begin{bmatrix}\frac{1}{a}+\Phi_{11} & \Phi_{12} & \Phi_{13} & . & \Phi_{1n}\\ \Phi_{21} & \frac{1}{a}+\Phi_{22} & \Phi_{23} & . & \Phi_{1n}\\ \Phi_{31} & \Phi_{32} & \frac{1}{a}+\Phi_{33} & . & \Phi_{3n}\\ . & . & . & . & .\\ \Phi_{n1} & \Phi_{n2} & \Phi_{n3} & . & \frac{1}{a}+\Phi_{nn}\end{bmatrix} \quad (12)$$

where

$$\Phi_{ij} = \phi_i(\zeta)\phi_j(\zeta), \qquad \phi_i(1)\phi_j(1) = -4 \ (i+j = odd)$$
$$4 \ (i+j = even)$$

$$\int_0^1 \phi_i(\xi)\phi_j(\xi)d\xi = \delta_{ij} \quad (13)$$

In equation (12), a is ratio of the added mass(M) to total mass(mL) of tube;

$$a = \frac{M}{mL} \quad (14)$$

2.3 Exact Solution for a Simple Model

Exact Solution for the system with added mass, M, at the free end was developed by Sim and Park [7]. In order to derive constant coefficient of fluid-elastic instability experimentally, a cantilever beam was considered as shown in Fig. 2(a). Acrylic flexible tube of small diameter is connected to added mass mounted at the tube end. Flow-induced vibration of tube can make in a relatively small flow, because of the flexible tube of small diameter. At the free end of the tube, an accelerometer was mounted to investigate the effect of the added mass on eigen-frequency. Because of a relatively short length of tube, rotating inertia and transverse shear stress can be ignored. As a result, equation of motion is expressed as follows;

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \quad (15)$$

Using separation variable mehod, solution of motion is defined in terms of axial variation(x) and time(t);

$$y(t,x)| = Y(x)q(t) \quad (16)$$

As a result, equation of motion can be separated into two equations, as follows;

$$\frac{d^2}{dx^2}\left(EI\frac{d^2Y(x)}{dx^2}\right) - m\omega^2 Y(x) = 0,$$

$$\frac{d^4Y(x)}{dx^4} - \lambda^4 Y(x) = 0 \quad (17)$$

where eigenvalue(λ) is calculated as

$$\lambda^4 = m\omega^2/(EI) \quad (18)$$

The solution of the mode shape is defined as

$$Y(x) = C_1\cos\lambda x + C_2\sin\lambda x + C_3\cosh\lambda x + C_4\sinh\lambda x \quad (19)$$

where the coefficients, C_j, are obtained by considering the following boundary conditions for clamped-free beam.

$$Y(x)|_{x=0} = 0, \qquad \left.\frac{dY(x)}{dx}\right|_{x=0} = 0,$$

$$\left.\frac{d^2Y(x)}{dx^2}\right|_{x=L} = 0, \qquad \left.\frac{d^3Y(x)}{dx^3}\right|_{x=L} = -\frac{M\cdot\omega^2}{EI}Y(L) \quad (20)$$

Characteristic equation of the system with added mass at the free end is expressed as

$$(1+\cos\beta_k\cosh\beta_k) + \frac{\beta_k M}{Lm}(\cos\beta_k\sinh\beta_k - \sin\beta_k\cosh\beta_k) = 0 \quad (21)$$

Mode shape for each mode can be calculated by

$$\phi_k(x) = -\cos(\beta_k x/L) + \sigma_k\sin(\beta_k x/L) + \cosh(\beta_k x/L) - \sigma_k\sinh(\beta_k x/L), \quad (22)$$

where $\sigma_k = (\cos\beta_k + \cosh\beta_k)/(\sin\beta_k + \sinh\beta_k)$,

Circular frequency can be expressed as

$$\omega_k = \beta_k^2\left(\frac{EI}{mL^4}\right)^{0.5} \quad (23)$$

2.4 Equivalent Spring Model.

Fig. 2(a) shows that the statics deflection, δ_{st},of a cantilever beam is due to the mass M attached to its free end. The equivalent spring system is shown in Fig. 2(b) The system will be applicable, if the cantilever is of negligible mass and M is small in size compare with length L. The static deflection due to the concentrated force Mg at the free end can be calculated by

$$\delta_{st} = \frac{MgL^3}{3EI} \quad (24)$$

where EI is the flexural stiffness of the beam. The equivalent spring constant, k_{eq},is defined as force per unit deflection.

$$k_{eq} = \frac{Mg}{\delta_{st}} = \frac{3EI}{L^3} \quad (25)$$

Considering the equivalent system, the circular frequency for the first mode is expressed as

$$\omega_1 = \sqrt{\frac{k_{eq}}{M}} = \sqrt{\frac{3EI}{ML^3}} = \beta_1^2\sqrt{\frac{EI}{mL^4}} \quad (26)$$

from which, the eigen-frequency is defined as

$$\beta_1 = \left(\frac{3mL}{M}\right)^{0.25} \quad (27)$$

Fig. 3. Experiment apparatus for simple test.

2.5 Simple Test for Comparison with Analytical Results.

Simple test is performed for comparison with analytical results. As shown in Fig. 3, flexible tungsten tube of small diameter is fixed by vice at one end of tube. Added mass and spring can be mounted at the free end or a node point of a mode shape. The frequency of tube with/without added mass and spring is measured, using an accelerometer and FFT analyzer.

3. Discussion of Results

Using equation (10) based on Galerkin's method, we can calculate the eigen-frequency and mode shapes. The mode shapes for the clamped-free beam with/without the added mass at the free end($x=L$)are shown in Fig. 3. We can find difference between the results given for with/without added mass. With increasing the added mass, mode shapes close to the shapes of clamped-clamped beam. The numerical solution of eigen-frequency is compared with exact solution, given by Sim and Park [7], in Table 1.

Also, the eigen-frequency given by the equivalent spring method for the first mode, is attached the table for a comparison. It is shown that the eigen-frequency decreases with the added mass and the equivalent spring method is applicable for the first mode with the added mass at the free end($x=L$). We found that the difference between the analytical results and the numerical results is negligible, however ~1% at the highest mode. When the added mass is located at different positions- (a) free end, node points of (b) second mode & third mode, the eigenvalue problem is solved by using Galerkin's method. The mode shapes with an added mass, located at each position-(a) or (b), are shown in Fig. 4 ($M/(mL)$=1) and Fig. 5 ($M/(mL)$=100). Both cases are given for $EI/(mL^4)$=1. The eigen-frequencies for both cases are tabulated in Table 2. It is found that the eigen-frequency for a mode shape, when an added mass is located at its node point, is slightly changed, while the frequencies of the other mode shapes are changed. For an example, the eigen-frequency of the second mode is almost constant($\beta_k \approx 4.694$) with an added mass, which is

Table 1. Comparison of eigenfrequencies, β_k , for clamped-free beam with added mass at the free end

$\frac{EI}{mL^4}$	$\frac{M}{mL}$	β_1 Eq.(27)	Mode (Exact Solution-Sim and Park[7])					Mode (Galerkin's Method)				
			1	2	3	4	5	1	2	3	4	5
0.1	0		1.875	4.694	7.855	11.00	14.14	1.875	4.694	7.855	11.00	14.14
	0.1	2.340	1.723	4.400	7.451	10.52	13.61	1.723	4.400	7.458	10.55	13.69
	1	1.316	1.248	4.031	7.134	10.26	13.39	1.248	4.034	7.151	10.31	13.53
	10	0.740	0.736	3.938	7.076	10.22	13.36	0.736	3.941	7.094	10.27	13.50
1	0.1	2.340	1.723	4.400	7.451	10.52	13.61	1.723	4.400	7.458	10.55	13.69
	1	1.316	1.248	4.031	7.134	10.26	13.39	1.248	4.034	7.151	10.31	13.53
	10	0.740	0.736	3.938	7.076	10.22	13.36	0.736	3.941	7.094	10.27	13.50
10	0.5	1.565	1.420	4.111	7.190	10.30	13.42	1.420	4.113	7.205	10.35	13.55
	5	0.880	0.870	3.950	7.083	10.22	13.36	0.870	3.953	7.101	10.28	13.50
	50	0.495	0.494	3.929	7.070	10.21	13.35	0.494	3.932	7.089	10.27	13.50

Table 2. Eigen-frequencies, β_k, for clamped-free beam with added mass and spring at a nodal point, ζ.

$\frac{EI}{mL^4}$	$\frac{M}{mL}$	$\frac{kL}{M}$	ζ	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	0	0	w/o*M*	1.875	4.694	7.855	11.00	14.14
1	1	0	1	1.248	4.031	7.134	10.26	13.39
			0.783	1.460	4.694	7.839	11.21	15.57
			0.502	1.755	4.470	7.812	9.191	14.14
1	100	0	1	0.416	3.928	7.069	10.21	13.35
			0.783	0.517	4.694	7.832	11.31	17.08
			0.502	0.789	3.980	7.563	8.193	14.15
100	1	100	1	1.341	4.034	7.151	10.31	13.53
			0.783	1.508	4.694	7.832	11.31	16.98
			0.502	1.765	4.467	7.812	9.191	14.14
100	100	1	1	0.447	3.931	7.088	10.27	13.50
			0.783	0.534	4.694	7.831	11.31	17.08
			0.502	0.793	3.980	7.563	8.193	14.15
100	100	100	1	1.007	3.931	7.088	10.27	13.50
			0.783	1.016	4.694	7.831	11.31	17.08
			0.502	1.079	3.980	7.563	8.193	14.15

mounded at the node point of the second mode. As a result, the added mass *M* has no influence on the frequency of the corresponding mode. With increasing the added mass, the node points of all mode shapes intend to collapse to the point where the added mass is loaded. When the added mass is located at the free

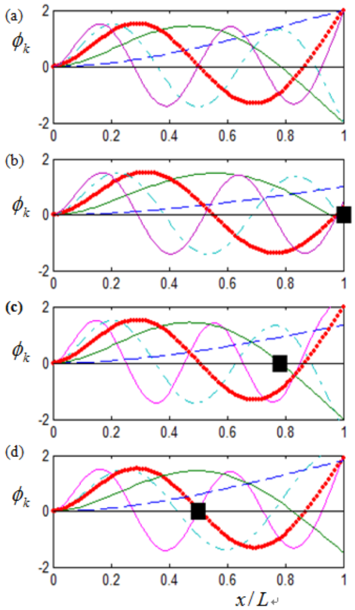


Fig. 4. Mode shapes, $\phi_k(x/L)$, of the clamped-free beam, $EI/(mL^4)$=1, (a) without added mass and with added mass, M/mL=1, at (b)$\zeta=$ 1, (c) $\zeta=$ 0.783, (d) $\zeta=$ 0.502.

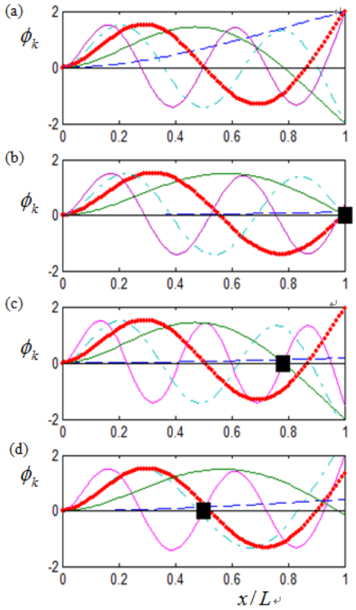


Fig. 5. Mode shapes, $\phi_k(x/L)$, of the clamped-free beam, $EI/(mL^4)$=1, (a) without added mass and with added mass, (M/mL=100), at (b)$\zeta=$ 1, (c) $\zeta=$ 0.783, (d) $\zeta=$ 0.502.

end, the free end becomes node points of all mode shapes with increasing the mass, as shown in (b) of Fig. 4 & 5. In other word, there is no displacement at the corresponding node point where a heavy added mass is loaded. The heavy added mass has a role of support on the flexible tube. For the clamped-free beam with added mass, *M*, Fig. 6 show that the eigen-frequency of the first mode decreases with increasing the added mass.

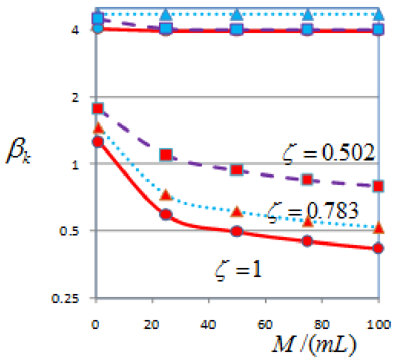


Fig. 6. Eigen-frequencies for clamped-free beam, $EI/(mL^4)$=1, with added mass, (M/mL=1).

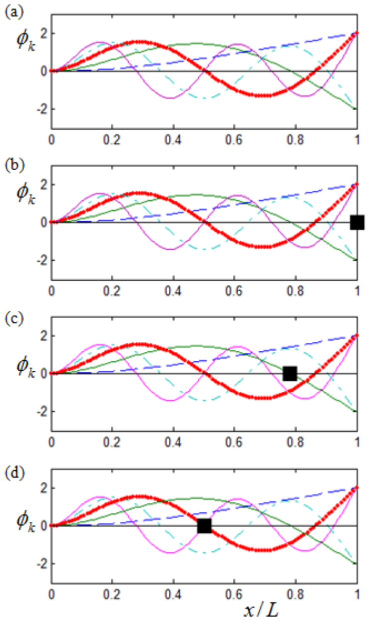


Fig. 7. Mode shapes, $\phi_k(x/L)$, of the clamped-free beam, $EI/(mL^4)$=1, (a) without spring and with spring, KL/M=100, at (b)$\zeta=$ 1, (c)$\zeta=$ 0.783, (d) $\zeta=$ 0.502.

Mode shapes for the clamped-free beam with spring and without added mass, are given in Fig. 7, to evaluate the stiffness effect on mode shape.

The stiffness is given by a spring loaded at the free end or at a node point (the second or the third mode). It is found the mode shapes do not influenced by the

Table 3. Eigen-frequencies, β_k, for clamped-free beam with added mass at free end or a nodal point, ζ.

	$\frac{EI}{mL^4}$	$\frac{M}{mL}$	ζ	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
	1	0	w/oM	1.875	4.694	7.855	11.00	14.14
GALERKIN	1	1	1	1.248	4.031	7.134	10.26	13.39
			0.783	1.460	4.694	7.839	11.21	15.57
			0.502	1.755	4.470	7.812	9.191	14.14
	1	100	1	0.416	3.928	7.069	10.21	13.35
			0.783	0.517	4.694	7.832	11.31	17.08
			0.502	0.788	3.980	7.563	8.193	14.15
ANSYS	1	1	1	1.247	4.030	7.132	10.25	13.39
			0.783	1.426	4.693	7.275	10.01	13.60
			0.502	1.696	3.778	7.853	9.759	14.13
	1	100	1	0.416	3.927	7.068	10.21	13.35
			0.783	0.499	4.693	7.018	9.803	13.52
			0.502	0.693	3.167	7.853	9.426	14.11

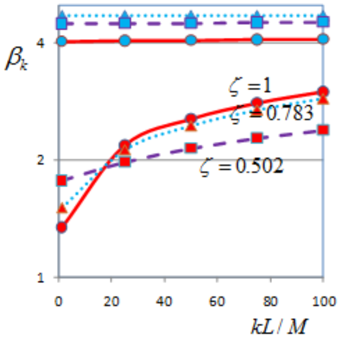


Fig. 8. Eigen-frequencies for clamped-free beam, $EI/(mL^4)$=1 & M/mL=1 with stiffness KL/M .

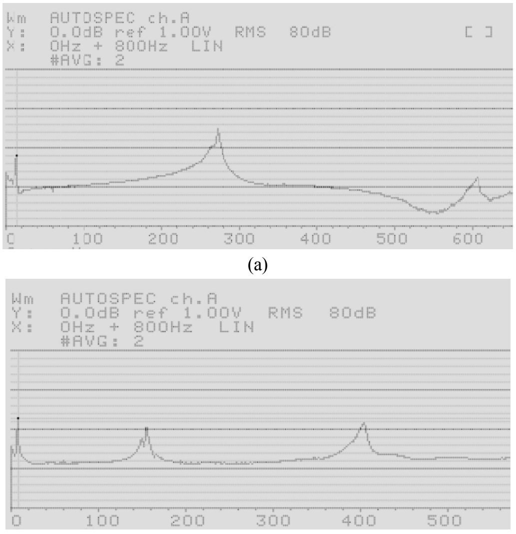


Fig. 9. Power spectral density, given by FFT analyzer, for a clamped-free beam with added mass (M/L=5) located at free end, $\zeta=$ 1; (a) $EI/(mL^4)$=380 (b) $EI/(mL^4)$=92.

stiffness. However, as shown in Fig. 8 for clamped-free beam, the eigen-frequency of the first mode increases with increasing the stiffness. Eigen-frequencies obtained by Galerkin's method and a commercial code(ANSYS) for clamped-free beam with added mass at free end or a nodal point are tabulated in Table 3. Good agreements between the results are shown. Experimental and analytical results of frequencies, $f_n=\omega_n/(2\pi)$, for clamped-free beam with added mass(M/mL=5) and spring at a free end or a node point, $\zeta = 1$ or 0.783, are shown in Table 4. Power spectral density, given by FFT analyzer, for a clamped-free beam with added mass (M/mL=5) located at free end, $\zeta = 1$, are illustrated in Fig. 9; (a) $EI/(mL^4)$=380 and (b) $EI/(mL^4)$=92. The values of experimental results are lower than those of analytical results. The main reason is due to the surface contact at the mounting point of the added mass and the spring. Analytical results are based on the point contact at the mounting point. The results are same trends with added mass and spring. For a example, the frequencies become lower with increasing the added mass and the frequencies increase with the spring force.

4. Conclusion

Galerkin's methods, as a numerical method to solve eigenvalue problem for a clamped-free beam, is introduced. The beam is subjected by added mass at the free end or a node point of mode shapes. To avoid tube failures in a heat exchangers due to excessive vibration of the beam, it is required to evaluate the eigen-frequency at design stage. Exact solution of the eigen-value problem has been developed by shown by Sim and Park [7], when the added mass is located at the free end. The eigen-frequencies obtained by the numerical method are compared with the results given by the simple analytical and experimental results, to show the availability of Galenkin's method.

The mode shapes and eigen-frequency for the clamped-free beam, are evaluated. It is shown that the eigen-frequency decreases with the added mass. Also, it is found the mode shapes do not influenced by the stiffness. However, the eigen-frequency of the first mode increases with increasing the stiffness. It is found that the eigen-frequency of which the added mass located at its node point are slightly changed, while others are change. With increasing the added mass mounted at the node point of one mode shape, all node points become to collapse to the point where the added mass is located. In other word, it is expected that the heavy added mass has a role of support on the flexible tube.

The eigen-frequencies obtained by the present numerical method are good agreement with approximate analytical results and experimental results given by the simple models and the simple test, respectively. As a result, the present Galerkin's method is applicable to estimate the eigen-frequency of the cantilever beam restrained with added mass and spring at free end or a node point. The eigen-frequency, for the system with the added mass mounted at the free end, can be calculated by the approximate analytical method given by Sim and Park [7] with more or less accuracy. As future works, it is required to perform precise experiments with point contact at the mounting place of the added mass and the spring.

References

[1] M. J. Pettigrew, and D. J. Gorman, "Vibration of Heat Exchange Components in Liquid and Two-phase Cross-Flow," Proceedings of the B.N.E.S. *Conference on Vibration in Nuclear Plant*, Keswick, U.K., Paper 2:3; also, Atomic Energy of Canada Limited Report AECL-6184, 1978.

[2] M. J. Pettigrew, Y. Sylvestre, and A. O. Campagna, "Vibration Analysis of Heat Exchanger and Steam Generator Designs," Nuclear Engineering and Design, vol. 48, pp. 97-115, 1978. DOI: http://doi.org/10.1016/0029-5493(78)90211-X

[3] R. D. Blevins, "Flow-Induced Vibration," Second Edition, Van Nosrtrand, New York, 1990.

[4] M. J. Pettigrew and C. E. Taylor, "Vibration Analysis of Shell-and-tube Heat Exchangers; An Overview- Part 2: Vibration Response, Fretting-wear, Guidelines", *Journal of Fluids and Structure*, vol. 18, pp. 485-500, 2003. DOI: http://doi.org/10.1016j.jfluidstructs.2003.08.008

[5] M. J. Pettigrew, C. E. Taylor, "Damping of Heat Exchanger Tubes in Two-Phase Flow: Review and Design Guidelines," ASME *Journal of Pressure Vessel Technology*, vol. 126, pp. 523-533, 2004. DOI: http://doi.org/10.1115/1.1806443

[6] H. I. Connors, "Fluidelastic Vibration of Tube Arrays Excited by Cross Flow," *Flow-Induced Vibration in Heat Exchangers*, ASME-WAM, NewYork, pp.42-56, 1970.

[7] W. G. Sim and M. Y. Park, Fluid-elastic Instability in a Tube Array Subjected to Two-Phase Cross Flow, *KSME-B,* vol. 33, no. 2, pp. 124-132, 2009.

DOI: http://doi.org/10.3795/KSME-B.2009.33.2.124

[8] M. J. Pettigrew, C. E. Taylor and B. S. Kim, "Vibration of Tube Bundles in Two Phase Cross Flow; Part 1 – Hydrodynamic Mass and Damping", *ASME Journal of Pressure Vessel Technology*, vol. 111, pp. 466-477, 1989.
DOI: http://doi.org/10.1115/1.3265705

[9] P. A. Feenstra, R. L. Judd and D. S. Weaver, "Fluidelatic Instability in a Tube Array Subjected to Two-phase R-11 Cross Flow", *Journal of Fluids and Structure*,. vol. 9, pp. 747-771, 1995.
DOI: http://dx.doi.org/10.1006/jfls.1995.1042

[10] M. P. Paidoussis, D. Mateescu and W. G. Sim, Dynamics and Stability of a Flexible Cylinder in a Narrow Coaxial Cylindrical Duct Subjected to Annular Flow, *ASME Journal of Applied Mechanics*, vol.57, pp. 232-240, 1990.
DOI: http://doi.org/10.1115/1.2888309

Woo-Gun Sim [Regular member]

- Sept. 1987 : McGill Univ., Canada, MS
- Nov. 1992 : McGill Univ., Canada, PhD
- Jan. 1992 ~ Dec, 1996: KAERI, Senior Researcher
- Mar. 1998 ~ current : Hannam Univ., Deptartment of Mechanical Engineering, Professor

<Research Interests>
Flow-induced vibration, two-phase flow and fluid dynamics