

# Uncoupled Solution Approach for treating Fluid-Structure Interaction due to the Near-field Underwater Explosion

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## 근거리 수중폭발에 따른 유체-구조 상호작용 취급을 위한 비연성 해석방법

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**Abstract** Because the water exposed to shock waves caused by an underwater explosion cannot withstand the appreciable tension induced by the change in both pressure and velocity, the surrounding water is cavitated. This cavitating water changes the transferring circumstance of the shock loading. Three phenomena contribute to hull-plate damage; initial shock loading and its interaction with the hull plate, local cavitation, and local cavitation closure then shock reloading. Because the main concern of this paper is local cavitation due to a near-field underwater explosion, the water surface and the waves reflected from the sea bottom were not considered. A set of governing equations for the structure and the fluid were derived. A simple one-dimensional infinite plate problem was considered to verify this uncoupled solution approach compared with the analytic solution, which is well known in this area of interest. The uncoupled solution approach herein would be useful for obtaining a relatively high level of accuracy despite its simplicity and high computational efficiency compared to the conventional coupled method. This paper will help improve the understanding of fluid-structure interaction phenomena and provide a schematic explanation of the practical problem.

**요약** 수중폭발로 인해 발생된 충격파에 노출된 유체(대부분 해수)는 유체장 내 압력과 속력 등의 물리적 변화에 따른 장력을 견딜 수 없으므로 캐비테이션(기포 또는 기공)이 발생하게 되고 이때 발생된 캐비테이션은 수중폭발의 연쇄 과정 중 구조물에 미치는 충격하중의 전달 환경을 변화시킨다. 폭발물과 구조물 간의 거리가 비교적 가까워 선체구조의 국부적 손상에 관심을 가지는 근거리 수중폭발연구에서 관심을 가지는 물리적 현상은 크게 3가지로 초기충격파 그리고 그것과 선체구조와의 상호작용, 국부 캐비테이션, 국부 캐비테이션 폐쇄 후 2차 충격파이다. 본 논문의 관심은 근거리 수중폭발에 따른 국소 캐비테이션이므로 수면과 해저로부터의 반사파는 고려하지 않는다. 유체와 구조에 관한 각각의 지배 방정식을 유도하고 이를 간단한 1차원 무한평판 문제에 적용, 수치적으로 해석하여 엄밀해와 비교해봄으로써 제안된 비연성 해석방법을 검증한다. 비연성 해석방법은 유체-구조 결합 해석방법보다 계산상 효율이 높으며 간단함에도 불구하고 상대적으로 높은 수준의 정확도를 얻을 수 있다는 점에서 유용하다. 본 논문을 통해 수중폭발과 같은 복잡한 물리적 상황에서의 유체-구조 상호작용 현상에 대한 이해와 실질적인 문제에 개념적 이해를 높이는 데 도움이 될 것이다.

**Keywords** : Fluid-Structure Interaction, Wave Approximation, Cavitation, UNDEX, PWA

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### 1. Introduction

When the compressive shock following the explosion under the surface of water arrives at the hull surface plate, structural boundaries require that a rarefactive wave be reflected. Due to both incident wave and reflective wave, the total pressure at the surface of hull-plate may drop to a certain low pressure. Since water cannot withstand appreciable tension, the surrounding water is cavitated and this cavitating water changes the circumstance of shock loading. The first cavitation locally occurs at the surface of the hull-plate as the primary shock wave is reflected from the hull-plate (local cavitation). The reflective wave from the water surface can create the cavitation with a thickness under the surface of water (bulk cavitation)[1]. When the total pressure again becomes compressive (positive value), the cavitation layer is destroyed then subsequently the successive shock is reloaded. It is called the secondary shock wave.

Once the cavitation occurs, the pressure and density relations are changed, and the bulk modulus becomes close to zero. The constitutive relation of fluid is considered as a bilinear one shown in Fig. 1[2,3]. Since the first use of this bilinear model, it has been widely employed in the region where the cavitation influences on the real functionality such as lubrication bearing, underwater explosion, earthquake in dam and so forth[4,5].

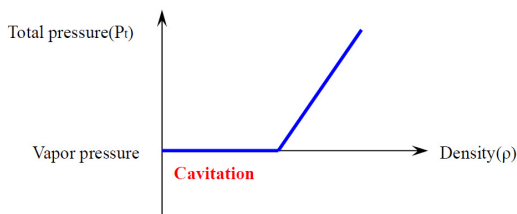
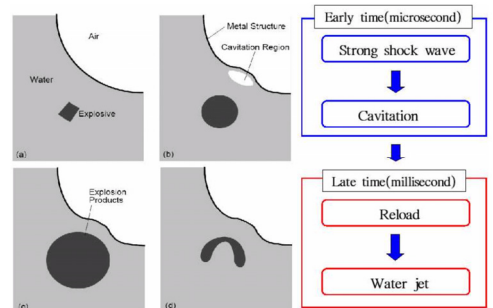
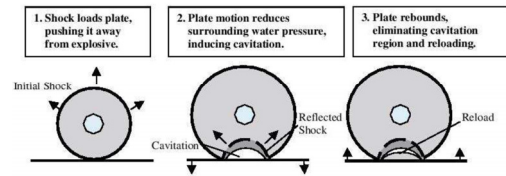


Fig. 1. The bilinear pressure-density relation[3]

Fig. 2 (a) shows the sequence of an explosion under the surface of water. The key features are shock wave, local cavitation, reloading, and bubble jetting. These are presented separately by two phases depending on the time scale.



(a) Sequential views of a near-field underwater explosion



(b) Mechanism of local cavitation on the surface

Fig. 2. Physics of near-field explosion and local cavitation[6]

To accurately model all of these aspects shown in Fig. 2, extensive methods development and large computing resources are essential. Due to these difficulties and complex physics, separate approach for various phenomenon shall be more efficient. This paper focuses on the local cavitation effect resulted from the interaction between the first(primary) shock wave and the surface of a structure. This shock wave and resulting local cavitation is dominant in the early-time since the energy distribution in underwater explosion is more than half as summarized in Table 1. Most of energy is consumed with the primary shock wave.

Since the main concern of this paper is the local cavitation due to the near-field underwater explosion, the water surface and the sea bottom reflection waves are not considered here.

Table 1. Energy consumption of a series of shock waves[1]

Loading	Energy consumption (%)
Primary shock wave	53
Second pulsating wave	31
Third pulsating wave	10

## 2. Equation of motion for the problem

For numerical study of this problem, one equation is required for the structure and the other equation for the fluid. To implement the interaction between the structure and the fluid, we need to set up equations with proper assumptions and then solve the equations sequentially. If the fluid is assumed as to be compressible and energy-dependent and the structure is assumed to be incompressible and adiabatic, we should solve a set of fluid equations (continuity, momentum, and energy equation) and single structure equation in case of a one-dimensional problem. There are many analytic approaches introduced in the community of explosion study: Taylor plate, Schechter/Box plate, Snay/Christian plate, Bleich/Sadler plate, etc[7]. However, most of existing methods do not allow the cavitation inception in the process within the set of equation since the water is modeled as a linear fluid. Some methods can obtain mathematically analytic solutions using the method of characteristics. The method of characteristics considers partial differential equations on a suitable hypersurface so that the solution can not be obtained with a simple mathematics. Numerical approach using engineering analysis theories and tools requires a lot of efforts to model the fluid and the structure respectively and is difficult to obtain the reasonable results without time discretization and meshing-related errors. Extensive method testing and development, fine-tuning of numerical methods and large computing resources are essential[5-7].

The uncoupled solution method is introduced to save such numerical efforts and likely errors. The key of an uncoupled solution method is to uncouple the structure and fluid equation by the wave approximation. Various approximations on the pressure field such as plane wave approximation (PWA), virtual mass approximation (VMA), and double asymptotic approximation (DAA)) have introduced. The PWA is widely used in cases where the wavelength is short. That is, in the early time near-field underwater explosion, the PWA can provide appropriate result enough in accuracy[3, 8]. This paper considers the PWA to approximate the fluid pressure following the near-field underwater explosion.

Let us consider a simple one-dimensional infinite plate. For a pressure field, the PWA is used to represent the relation between the pressure field  $p$  and corresponding velocity  $v$  as  $p = \rho cv$ . The terms  $\rho$  and  $c$  are density and speed of sound for medium. The key of the approach is to simplify the fluid equation by the wave approximation then put the quantity (i.e. pressure loading) into the structure equation as the surface traction force. The equation of structural motion is given as

$$M \frac{dv}{dt} = f \quad (1)$$

where  $v$  is the particle velocity,  $M$  is the plate mass, and  $f$  is the external load obtained from shock loading. Since the external load is equal to the total pressure loading  $p_T$  consisting of incident pressure  $p_i$  and reflected pressure  $p_r$ , Eq. (1) can be rewritten as

$$M \frac{dv}{dt} = p_T \quad (2)$$

The boundary conditions and the initial conditions in terms of velocity are defined as

$$\begin{aligned} v(0, t) &= v_i + v_r \\ v(x, 0) &= 0 \end{aligned} \quad (3)$$

where velocity term  $v_i$  and  $v_r$  are the particle velocity of the incident wave and reflective wave, and the plate initially sets rest.

Substituting the PWA relation into Eq. (3) gives

$$v(0, t) = \frac{p_i - p_r}{\rho c} \quad (4)$$

where  $\rho c$  is the acoustic impedance which is the ratio of pressure to flow. Equating Eq. (4) about  $p_r$  gives

$$p_r = p_i - \rho c v(0, t) \quad (5)$$

Substituting Eq. (5) into Eq. (2) then putting it in Eq. (1) becomes

$$M \frac{dv}{dt} + \rho c v = 2p_i \quad (6)$$

From Eq. (6), it is realized that the sum of pressures in terms of the incident and reflect wave causes substantially a pressure doubling[1-3].

In order to find the pressure field, the incident pressure component is defined as in [1-3]

$$p_i(t) = p_o e^{-t/\theta} \quad (7)$$

where  $p_o$  is the magnitude of initial shock and  $\theta$  is the decaying coefficient, respectively. If  $p_o = 0.786 MP_a$  and  $\theta = 0.0022$  sec. set as in [3], the incident pressure is exponentially decayed with function of time  $t$  and the decaying coefficient  $\theta$  shown as Fig. 2.

Substituting Eq. (7) into Eq. (6) then solving mathematically gives

$$v(0, t) = \frac{2p_o\theta}{M(\beta-1)} \left( e^{-\frac{\beta t}{\theta}} - e^{-\frac{t}{\theta}} \right) \quad (8)$$

where the term  $\beta$  is  $\rho c \theta / M$ .

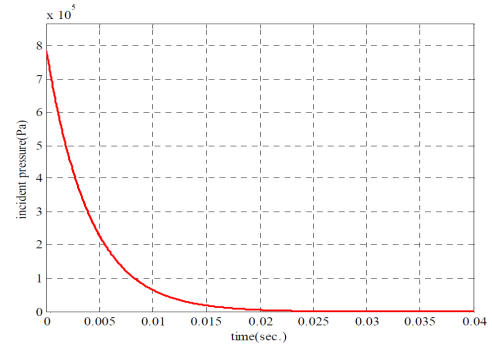


Fig. 3. Time-history of incident shock loading

With Eq. (8), the expression of the reflected wave  $p_r$  is rewritten as

$$p_r = p_o \left[ e^{-\frac{t}{\theta}} - \frac{2p_o\theta}{(\beta-1)} \left( e^{-\frac{\beta t}{\theta}} - e^{-\frac{t}{\theta}} \right) \right] \quad (9)$$

where the first term is the incident shock and the second term is the reflected wave. Thus, the total pressure  $p_T$  is given as

$$\begin{aligned} p_r &= p_i + p_r + p_a + mg \\ &= 2p_o \left[ e^{-\frac{(t+x/c)}{\theta}} - \frac{\theta}{(\beta-1)} \left( e^{-\frac{\beta(t-x/c)}{\theta}} - e^{-\frac{(t-x/c)}{\theta}} \right) \right] + p_a + mg \end{aligned} \quad (10)$$

where  $p_a$  and  $mg$  are the ambient pressure and the gravitational component respectively[1]. Eq. (6) is an analog of mass-damper system and the acoustic impedance  $\rho c$  acts as a damper. That is, it is a linear-viscous dashpot attached to the mass  $m$  as in Fig. 4.

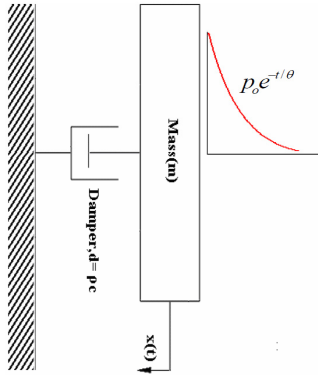


Fig. 4. The free-body diagram of fluid-structure interaction

### 3. Solution approach

The equation of motion is a form of the first-order differential equation with the damping term. The equation is generally solved by numerical method. However, in the cavitation phase, since the cavitation layer separates the plate with the surrounding water, Eq. (6) cannot be applied. That is, the dashpot is detached from the mass  $m$ . Hence the modified equation must be introduced to consider such nonlinear behavior. The equation of motion should be computed in the linear interaction phase, cavitation phase, and the reloading phase, sequentially.

#### 3.1 Linear interaction phase

The system is schematically shown in Fig. 4. In the absence of cavitation layer, Eq. (6) is directly applied as

$$M \frac{dv}{dt} + \rho c v = 2p_i$$

Following defining the incident pressure, the time-dependent Eq. (6) is numerically integrated to compute the velocity with respect to time.

#### 3.2 Cavitation phase

When the total pressure becomes close to a vapor pressure (or negative pressure), the surrounding water is cavitated. The resulting cavitating layer formed at the hull plate surface separates the plate and the shock loading temporarily. Thus, Eq. (6) can no longer be applied for this phase. Since the cavitation layer is entirely at vapor pressure, separation may take place[2]. The separation leads to the detachment of dashpot from the mass. The mass is not affected by the pressure field due to the shock loading, but only affected by the gravitational force  $mg$ . Hence the cavitation layer created at the plate surface decelerates the plate motion only by the gravitational force[3].

$$M \frac{dv}{dt} = -(p_a + mg) \quad (11)$$

Eq. (11) is applied when the total pressure is less than or equal to the vapor pressure.

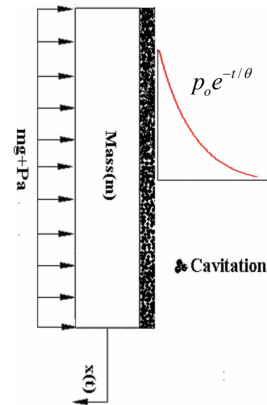


Fig. 5. The free-body diagram of fluid-structure interaction in the cavitation phase

This phase can also be separated into the separation and the fall phase mentioned by F.L.Dimaggio[3]. In the fall phase the plate is decelerating and the plate motion will be affected by the added mass effect, viscous

friction, etc. This work does not include such effects but it is a considerable subject in the underwater explosion study.

### 3.3 Reloading phase

As time continues, the total pressure in a cavitation layer becomes positive so that the cavitation layer is destroyed. Now, the dashpot is again attached to the mass and Eq. (6) can be used again. However, whenever the total pressure becomes a vapor pressure or less again, or enters into the cavitation phase, Eq. (11) must be considered.

### 3.4 Numerical example

I.S.Sandler[8] provided the example of analytic solution useful for numerically verifying another solution approach. The solution for the structural response in Eq. (6) and Eq. (11) are obtained by the numerical time integration scheme. As initial conditions, the displacement and the velocity of mass are defined as 0.0 at time  $t = 0.0$ sec. For the reflection of shock loading, the boundary condition is defined as the sum of the incident and reflective particle velocity. The many of numerical time integration schemes are typically applicable: Euler-explicit, Euler-implicit and Runge-Kutta. Runge-Kutta methods are commonly used in many applications since it provides very accurate and stable solutions, and avoids the need for higher derivatives. Runge-Kutta (RK) methods for numerical integration of the ordinary differential equations are also popular because of their simplicity and efficiency. Here, the fourth-order RK numerical integration scheme is employed to compute the solution with  $\Delta t = 0.0001$ sec. Although the fourth-order RK method requires four evaluations per step, it gives more accurate answers than others with larger time step sizes.

Let consider the standard integral form as

$$\{\dot{v}(t)\} = \{g(t, v(t))\} \quad (12)$$

Here let  $v_1 = \dot{x}$  and  $v_2 = x$ . Thus both Eq. (6) and Eq. (12) can be rewritten by

$$\begin{aligned} m\ddot{x} + \rho c\dot{x} &= 2p_i(t) \text{ or } m\dot{v}_1 + \rho cv_1 = 2p_i(t) \\ \dot{v}_1 &= \frac{-\rho cv_1 + 2p_i(t)}{m} \text{ and } \dot{v}_2 = \dot{x} = v_1 \end{aligned} \quad (13)$$

$$\begin{aligned} m\dot{v}_1 &= -(p_a + mg) \\ \dot{v}_1 &= \frac{-(p_a + mg)}{m} \text{ and } \dot{v}_2 = \dot{x} = v_1 \end{aligned} \quad (14)$$

The time-dependent profile of total pressure at the surface of plat is shown as

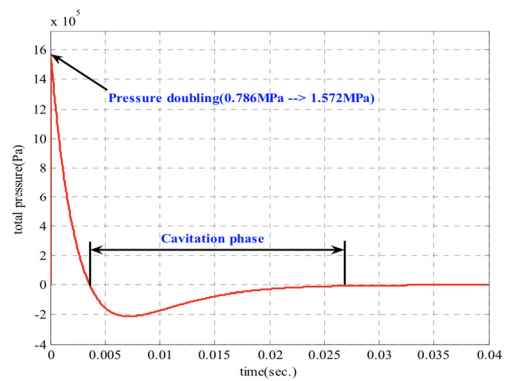


Fig. 6. The time-dependent profile of total pressure at the surface of plate

The peak pressure at very early time is about 1.572MPa which is a double of the pressure magnitude of the incident wave.

Fig. 7 describing the plate velocity shows the early shock loading with the pressure doubling, a long cavitation period, and the reloading after the closure of cavitation layer as in [3]. It provides good agreement between reference analytic solution and the uncoupled numerical solution. Linear solution shows no cavitation characteristics as we expected.

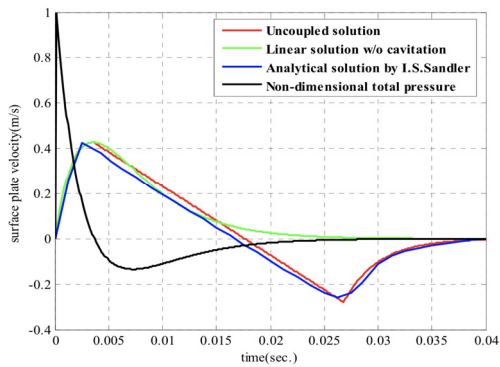


Fig. 7. The time-dependent profile of the plate velocity

During the cavitation phase, the velocity profile explains the deceleration of mass only affected by the sum of ambient pressure and its weight. It is seen that the solution conducted by this uncoupled approach agrees relatively well with that of original paper[3] and predicts all three phases of shock loading well. However, the inception time and closure time of cavitation at the plate surface was not exactly coincident with the assumed criteria. In fact, the criterion for the cavitation closure was set to  $-6kPa$  but the cavitation inception was  $0.0Pa$  in the numerical code since the total pressure was never recovered to the positive value in this example setup. The difference may be due to the existence of ambient pressures and gravitational force acting on the plate. The results are compared in Table 2.

Table 2. Comparison between analytical and numerical solutions: plate velocity (m/s) and time (sec.)

Region	Analytical solution	Numerical solution	Difference
Linear-Cavitation	0.426 m/s (0.0025 sec.)	0.457 m/s (0.0029 sec.)	0.031 m/s (0.0004 sec.)
Cavitation-Reloading	-0.253 m/s (0.0260 sec.)	-0.276 m/s (0.0267 sec.)	0.023 m/s (0.0007 sec.)

The difference in the slope of the numerical problem with slope in the analytical problem may result from the fact that during the fall

phase, the approximation does not allow the falling effect of the plate in the surrounding water region[3].

## 4. Conclusion

The physical explanation of cavitation and its effect on the plate are summarized. The benchmark example is computed by the suggested uncoupled method and compared with the analytical solution. Although the author did not find the exact information of the cavitation inception and closure time, the uncoupled solution method can be useful to compute all three shock loading phases following the underwater explosion which can be expanded to other practical methods such as the finite element method. Advantages and disadvantages of this uncoupled approach compared with previous solution approaches are summarized in Table 3.

Table 3. Pros and cons of the uncoupled solution method

Pros	<ul style="list-style-type: none"> <li>- Easy separation of complex phenomenon</li> <li>- Useful to describe the motion of plate exposed to the shock loading with less effort</li> <li>- Quick delivery of approximated solutions for heuristic discovery</li> </ul>
Cons	<ul style="list-style-type: none"> <li>- Difficult application to higher dimensional problem</li> <li>- Slight deviation with analytic solutions"</li> </ul>

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⟨Research Interests⟩

Systems Engineering Analysis, Data Science, Concept Design, Set-Based Design, Design Optimization