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Discrete Sizing Optimization of Truss Structures Using Continuous Optimization and Machine Learning Tools

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연속최적화와 기계학습도구를 이용한 트러스 구조물의 이산치수 최적화

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Abstract This research proposes a series of design procedures for discrete sizing optimization of truss structures. Most discrete sizing optimization methods for truss structures adopt stochastic approaches using metaheuristic algorithms. However, such methods involve many structural analyses until they find a discrete optimal solution, which is expensive. The primary motivation of this research is to suggest a discrete design while reducing the number of structural analyses as many as possible. First, the structural optimization software GENESIS performs sizing optimization in a continuous design space using proven techniques. This provides an excellent optimal solution, and the proposed method is applied while assuming that the discrete optimal solution exists near the continuous optimum design. For discrete sizing optimization of a truss structure, approximate models are generated near the continuous optimum point using the scikit-learn-one capability of machine learning libraries, leading to a simple optimization problem. Then, the python library, PyGAD, is used to obtain a discrete optimal solution. Compared with existing methods, this research provides discrete designs requiring only 0.38-52.0% of the number of structural analyses performed in other studies.

요 약 본 연구에서는 트러스 구조물의 이산 치수 최적화를 위한 일련의 설계 절차를 제안하고 있다. 기존 트러스 구조물의 이산 치수 최적화 방법의 대부분은 메타휴리스틱 알고리즘을 사용한 통계적 접근 방식을 채택하고 있다. 그러나 이러한 방법은 이산 최적해를 찾을 때까지 많은 해석을 요구하므로 고가이다. 본 연구의 주된 동기는 구조해석의 수를 최대한 줄이면서 이산설계를 제안하는데 있다. 첫째, 잘 개발된 구조 최적화 기법이 포함된 상용 소프트웨어인 GENESIS를 이용하여 연속설계 공간에서 최적해를 산출한다. 연속설계공간에서의 최적해는 우수한 해를 제공한다. 그 다음, 이산 최적해가 연속 최적해 부근에 있다고 가정하여 제안하는 방법을 적용한다. 트러스 구조물의 이산 치수최적화를 위해 머신러닝 라이브러리 중 하나인 scikit-learn에 내장된 기능을 사용하여 근사모델을 연속 최적해 근처에서 생성하여 간단한 최적화 문제로 변환한다. 그런 다음 파이썬 라이브러리인 PyGAD를 사용하여 이산 최적해를 산출한다. 본 연구는 기존의 메타휴리스틱 알고리즘을 연계한 접근법에 비해 0.38 ~ 52.0%의 구조해석 횟수로서 이산설계를 제공하고 있다.

Keywords : Truss Structure, Sizing Optimization, Discrete Design, Continuous Optimization, Machine Learning

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1. Introduction

The sizing optimization problem of truss structures should be treated in a discrete design space rather than a continuous design space if design variables are selected from a standard specification or design codes. The discrete sizing optimization problem has non-convex and combinatorial characteristics. Therefore, structural optimization techniques in continuous design space cannot be directly applied even though it was well developed. Most recent studies on discrete structural design have adopted stochastic approaches such as metaheuristic algorithms[1]. Since such methods do not use gradient information to find an optimal solution, they can be usefully applied to optimization problems, including multi-modal function, nonlinear function with noises, discontinuous function, etc.[2]. However, these methods inevitably require a lot of function calculations compared with gradient -based algorithms to find an optimal solution because the direct link between metaheuristic algorithms and structural analysis triggers numerous structural analyses. In addition, the quality of an optimal solution may depend on the parameters defined in the algorithm. Contrary to stochastic approaches, the deterministic structural optimization approach in continuous design space has the advantage of determining the optimal solution without many structural analyses because it uses a mathematically clearly defined sensitivity analysis, even though it provides a local optimum. This research aims to find a discrete solution by utilizing the advantage of continuous optimum design while reducing the number of structural analyses.

Although discrete sizing optimization of truss structures is a classic topic, there are ongoing studies on different methods due to the difficulty in solving the problem. Most recent studies on the discrete design of truss structures are looking for the optimal solution by applying metaheuristic algorithms. The already proven and well-developed metaheuristic methods that simulate natural laws or natural phenomena have been used directly or modified to the discrete design of truss structures. Such algorithms include SA[3], PSO[4], MBA[5,6], GA[7], ESASS[8], MCSS[9], SSA[10], ADS[1], DE[11], HHS[12], ACCS[13], NMA[14], HDA[15], etc. Although these algorithms cannot mathematically guarantee that they will find the global optimum, they try to find it in a stochastic or heuristic way. Still, the discrete optimal solution can be obtained through trial and error by setting appropriate parameters related to each algorithm. However, such a method requires many structural analyses to find the discrete optimum design.

This research presents a series of design procedures to overcome the disadvantage caused by directly linking metaheuristic algorithms and structural analysis in discrete sizing optimization. A design process for discrete sizing optimization of truss structures presented in this research is as follows: First, it is assumed that a discrete optimum exists around a continuous optimum. Second, the optimal solution is calculated using well-developed structural sizing optimization techniques in continuous design space defined within a design range using GENESIS[16,17], a structural optimization software. Third, two to four discrete candidates for each design variable are selected near the continuous optimum. The new lower and upper bounds for the discrete design are specified considering the range of discrete values determined in the previous stage. Sample points are generated within this design range according to the number of design variables using the space-filling method for generating approximate models. After performing FE analysis on each sample point, the approximate models composed of polynomials are created. In this process, the polynomials are constructed using the class of scikit-learn[18], one of the machine learning libraries. Once approximate models in polynomials are built, any existing metaheuristic algorithm can be applied to the optimization problem made of the polynomials. Since all the objective and constraint functions defined in the optimization problem are replaced with the polynomials, finding the optimal solution becomes a simpler and more efficient process. While most existing studies adopt the stochastic approaches to implement discrete designs, this research is close to the deterministic approach.

The suggested design method provides feasible discrete designs while significantly reducing the number of structural analyses compared with existing studies. The efficiency of the proposed design process is measured through three standard test problems of 10-bar, 25-bar, and 72-bar truss structures, and the results are compared with existing methods. The design process is implemented in Python, and PolynomialFeatures, a class of the machine learning library scikit-learn[18], and Python library PyGAD[19] are used.

2. Design process

2.1 Determination of continuous optimum design

The structural sizing optimization formulation for the test problems is presented as follows:

$$Minimize \quad W(\boldsymbol{x}) \tag{1}$$

$$s.t. -\sigma_{all} \leq \sigma_i(\boldsymbol{x}) \leq \sigma_{all} \, (i=1,\ldots,n_e) \tag{2}$$

$$-\delta_{all} \leq \delta_{ij}(\boldsymbol{x}) \leq \delta_{all} \, (i: node, j=x,y, \text{or } z) \tag{3}$$

$$-\boldsymbol{x}_L \leq \boldsymbol{x} \leq \boldsymbol{x}_U \tag{4}$$

where W, $\boldsymbol{x}$, $\boldsymbol{x}_L$, $\boldsymbol{x}_U$, σ, δ, n_e, and subscript *all* represent weight, design variable vector, its lower and upper bounds, stress, displacement, number of elements, and allowable value, respectively. The structural sizing optimization to find the optimum design with continuous variables is performed by using GENESIS. The optimization by GENESIS mostly requires only 10 or so detailed FE analyses to achieve the optimum, even when there are huge numbers of design variables and constraints[17].

2.2 Selection of discrete candidates

A discrete optimal solution is assumed to exist near the continuous optimum obtained by the structural sizing optimization techniques and deterministic algorithm. The location of the continuous optimum point of each design variable, corresponds to one of the five types in Fig. 1. The circle in Fig. 1 means an element of a discrete set. If the location of x_i^* exists between the lower and upper bounds as shown in Fig. 1(a), a total of four discrete values, two each, centering continuous optimum point from up to down, are selected as discrete candidate values. On the other hand, if x_i^* is located as shown in Fig. 1(b) or (c), a total of three discrete values, including the upper or lower bound, are selected as discrete candidate values. Finally, if x_i^* converges to the lower or upper bound, a total of two discrete values, including the lower or upper bound, are selected as discrete candidate values. When all discrete candidates are determined for

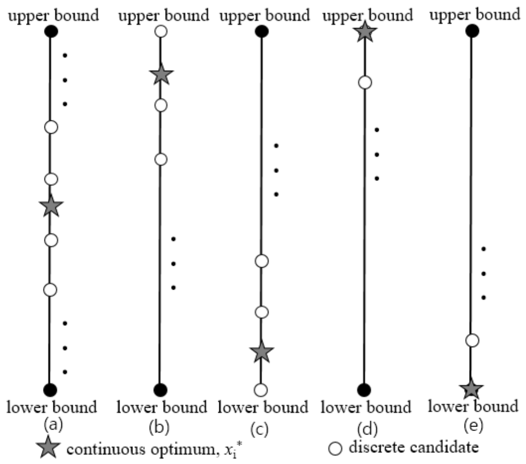

Fig. 1. Selection of a discrete candidate set

each design variable, new lower bounds, LB_d, and new upper bounds, UB_d, are defined from the discrete candidate set.

2.3 Suggested design procedure

The following design process is proposed for the discrete sizing optimization of truss structures as follows.

Step 1: An optimal solution is calculated using the structural optimization techniques in the continuous design space defined in Eq. (4). The optimum point obtained in this way is called the continuous optimum point. In this process, active constraints are extracted among the constraints to create efficient approximation functions, and GENESIS, a software that includes structural optimization techniques, is used.

Step 2: In the neighboring range of the continuous optimum point, discrete values are chosen as described in Section 2.2. A discrete candidate set is made for each design variable. Based on the discrete candidate sets for design variables, LB_d and UB_d for discrete design are defined.

Step 3: Sample points are created to generate the quadratic polynomials of the response values such as weight, displacement, and stress included in Eq. (1) to (3). The sample points are created between LB_d and UB_d using the Latin hypercube design, one of the space-filling methods. The number of sample points, n_s is determined as follows in consideration of number of design variables n, number of training data n_{tr}, number of test data n_{te}, and number of polynomial coefficients n_p.

$$n_{tr} = round(1.5n_p),\ n_{te} = round(0.2n_{tr}),\quad n_s = n_{tr} + n_{te} \tag{5}$$

$$n_p = 1 + 2n + \frac{n(n-1)}{2} \tag{6}$$

Then, FE analysis is performed for each sample point. The number of sample points means the number of FE analyses in this step.

Step 4: Using PolynomialFeatures, a class of Python library scikit-learn implementing machine learning algorithms creates the quadratic functions of responses in Eq. (1)-(3), but only the responses for displacement and stress included in the active constraints set in Step 1 are considered. Only sample points are used to generate the quadratic polynomials while sample points to calculate the *RMSE*. The *RMSE* to an i response, y_i is calculated as

$$RMSE_{si} \text{ or } RMSE_{di} = \sqrt{\frac{1}{n_{te}}\sum_{j}^{n_{te}}\left(y_j - \hat{y}_j\right)^2} \tag{7}$$

Step 5: A formulation of sizing optimization of the truss structure set in Eq. (1) to (4) is modified as follows:

$$Minimize \quad \widehat{W}(\boldsymbol{x}) \tag{8}$$

$$Subject\ to\ \hat{\sigma}_i(\boldsymbol{x}) \le \sigma_{all} - (iter-2) \times RMSE_{si} \quad (i = 1, \cdots, n_{as}) \tag{9}$$

$$\hat{\delta}_{ij}(\boldsymbol{x}) \le \delta_{all} - (iter-2) \times RMSE_{di} \quad (i = 1, \cdots, n_{ad}) \tag{10}$$

$$\boldsymbol{LB_d} \le \boldsymbol{x} \le \boldsymbol{UB_d} \tag{11}$$

where *iter*, n_{as}, and n_{ad} represent the number of outer iterations, the number of active stress constraints, and the number of active displacement constraints. This formulation approximates only the responses included in the active constraints extracted in Step 1. The displacements and stresses approximated by the polynomials in Eq. (9), (10) inevitably have errors with true values. Therefore, if the allowable values set in Eq. (2), (3) are used as they are, then the exact discrete optimal solution may not be obtained. To overcome that, the allowable values of the inequality constraints are relaxed or tightened by introducing *RMSE* values. The modified allowable values are indicated on the right-hand side of Eq. (9), (10) by reflecting *iter*

Table 1. Continuous optimal solutions due to starting points

Case	$\mathbf{x}^0$ (in^2)	$\mathbf{x}^*$ (in^2)	$w^*(lb_f)$
10-bar Case 1	1.62	32.168, 1.620, 23.375, 15.250, 1.621, 1.620, 8.427, 22.581, 21.575, 1.620	548.31
10-bar Case 2	5.00	32.097, 1.621, 23.389, 15.261, 1.620, 1.621, 8.419, 22.612, 21.582, 1.620	548.29
10-bar Case 3	33.0	32.128, 1.622, 23.514, 15.301, 1.620, 1.622, 8.454, 22.431, 21.579, 1.660	548.46
25-bar Case 1	0.10	0.100, 0.538, 3.400, 0.100, 1.828, 0.947, 0.431, 3.400	484.27
25-bar Case 2	1.50	0.100, 0.458, 3.400, 0.100, 1.879, 0.960, 0.463, 3.400	484.10
25-bar Case 3	3.40	0.100, 0.450, 3.400, 0.100, 1.886, 0.961, 0.466, 3.399	484.12
72-bar Case 1	0.20	1.887, 0.512, 0.100, 0.100, 1.269, 0.511, 0.100, 0.100, 0.524, 0.517, 0.100, 0.100, 0.156, 0.546, 0.411, 0.569	379.62
72-bar Case 2	2.00	1.884, 0.513, 0.100, 0.100, 1.268, 0.512, 0.100, 0.100, 0.524, 0.517, 0.100, 0.100, 0.156, 0.546, 0.411, 0.570	379.63
72-bar Case 3	3.20	1.887, 0.512, 0.100, 0.100, 1.271, 0.511, 0.100, 0.100, 0.523, 0.517, 0.100, 0.100, 0.157, 0.546, 0.411, 0.568	379.62

and *RMSE*. That is, for the active constraints determined in Step 1, the relaxed or tightened allowable values are determined by considering the *RMSE* for the responses and the *iter*. The allowable values become tighter as *iter* increases. The solution of the formulation of Eq. (8)-(11) can be infeasible, which is checked and filtered in step 7 through an FE analysis.

Step 6: Since all the responses in Eq. (8)-(11) are substituted with the polynomials, the desired outcome is achieved with less computing time regardless of which metaheuristic algorithm is used in calculating the discrete optimum point. In this research, PyGAD[19], an open-source Python library for implementing the genetic algorithm, is utilized to solve Eq. (8)-(11) in the discrete design space. As the values of parameters constituting the genetic algorithm increase, the probability of finding a global optimum point also increases. However, this process may result in extreme values of parameters that inevitably affect the calculation time. Although the formulation consists of simple polynomials, it may require numerous function calculations in the optimization process, taking a substantial amount of calculation time, thus undermining the purpose of this research. Therefore, appropriate values are assigned to the parameters included in PyGAD, and the genetic algorithms are repeatedly applied while changing the initial value k times to compensate for falling into a local optimum point.

Step 7: The k solutions performed in step 6 are sorted in ascending order based on the size of objective function values. Then, FE analysis is performed on the discrete design variables with the smallest objective function value. If the constraints in Eq. (2), (3) are satisfied, a final decision is made regarding the discrete optimal solution, and the design process ends. If any of the constraints of Eq. (2), (3) is violated due to the FE analysis, then the following combination of design variables is reviewed. As a result of the FE analysis, if it is found that a constraint not included in the active constraints is violated, it is added to the active constraints and returns to Step 5. If any feasible solution is not obtained up to number $j=k-1$, move to the next step.

Step 8: The case that any feasible solution is not found from Step 7 to the k-th iteration is due to the modified allowable values defined in Eq. (9), (10). Thus, the allowable values included in the second terms on the right side of Eq. (9), (10) are tightly adjusted. When the modified allowable values are set in Step 5, the design process proceeds to Step 6. Inclusion of the number of iterations representing the circulation of Step 5 to Step 7 is to relax or tighten the allowable values.

3. Test examples and results.

The problems of 10-bar, 25-bar, and 72-bar truss structures are representative examples used as test problems to benchmark and compare a discrete sizing optimization method of truss structures. The problems are solved by applying the suggested design process. The unit systems represented in existing studies are used as they are to compare the results of this research with those of existing studies. The results of the present and existing studies are compared with the optimum weight, *NSA*(number of structural analyses), *EET*(estimated elapsed time), and *CV*(constraint violation). In the comparison table, the weight in existing studies using stochastic approaches indicates the best weight, while the *NSA* in existing studies is the minimum number in the case of multiple trials. The *EES* of the present study is the total elapsed time up to the final decision, including the continuous optimal solution calculation time, FE analysis performing time for sample points, polynomial building time, and discrete optimal solution calculation time using PyGAD. In contrast, the *EES* of existing studies is only the estimated total calculation time of the FE analyses required for *NSA* by considering the FE analysis for one time in the computer with 2.20 GHz-2 Processor-CPU and 64GB-RAM. For each test problem, while changing the initial values three times, represented as Case 1, Case 2, and Case 3, the continuous optimum designs are obtained using GENESIS. The discrete optimal solutions are determined by applying the steps of the proposed design process for each local continuous optimum design. Table 1 summarizes the initial values, $\boldsymbol{x}^0$ and continuous optimal solutions, $\boldsymbol{x}^*$ for each test problem. In all examples, k is set to 10.

3.1 10-bar truss

The 10-bar truss structure[13-15], as shown in Fig. 2 has the material properties of density ρ =0.1 lb/in^3, and young's modulus E=10,000 ksi. The design variable is the cross-sectional area of each element, thus n=10, and for this sizing optimization problem, δ_{all} is 2.0 in, and σ_{all} is 25,000 psi. The discrete design variables are selected from the set, S={1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} in^2. From Table 1, it can be seen that the same optimal solution is obtained even if the initial values are changed. The discrete optimal solution is obtained from the first number of iterations (*iter*=1), which is compared with existing studies as shown in Table 2. The suggested discrete optimum design is the same as SA[3], aeDE[11], SSA[10], ADS[1], HHS[12], EFA[20], ACCS[13], and NMA[14], thus lies in the feasible region. In other studies, the weight of HPSO and MBA increases by 0.75% and 0.31%, respectively, and HDA provides an infeasible design. Looking at *NSA*, the *NSA* of this research is only 0.38~14.6% compared with ADS, which

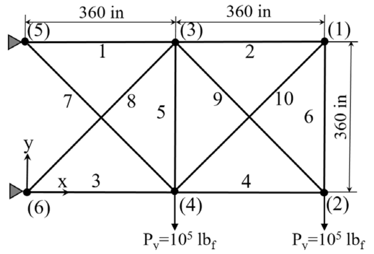

Fig. 2. 10-bar truss structure

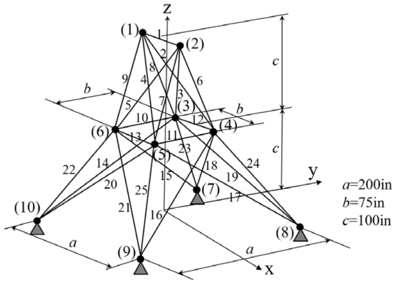

Fig. 3. 25-bar truss structure

Table 2. Comparison of discrete designs for the 10-bar truss

x^*(in^2)	HPSO[4]	SA[3]	MBA[6]	aeDE[11]	SSA[10]	ADS[1]	HHS[12]
x_1	30.00	33.50	30.00	33.50	33.50	33.50	33.50
x_2	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_3	22.0	22.90	22.90	22.90	22.90	22.90	22.90
x_4	13.50	14.20	16.90	14.20	14.20	14.20	14.20
x_5	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_6	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_7	7.97	7.97	7.97	7.97	7.97	7.97	7.97
x_8	26.50	22.90	22.90	22.90	22.90	22.90	22.90
x_9	22.00	22.00	22.90	22.00	22.00	22.00	22.00
x_{10}	1.80	1.62	1.62	1.62	1.62	1.62	1.62
W^*(lb$_f$)	5531.91	5490.74	5507.75	5490.74	5490.74	5490.74	5490.74
NSA	50,000	10,500	3,600	2,550	5,050	1,000	5,000
EET(s)	17,000	3,570	1,224	867	1,717	340	1,700
CV(%)	0	0	0	0	0	0	0

Table 2. (continued)

x^*(in^2)	EFA[20]	ACCS[13]	NMA[14]	HDA[15]	Present study		
					Case 1	Case 2	Case 3
x_1	33.50	33.50	33.50	33.50	33.50	33.50	33.50
x_2	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_3	22.90	22.90	22.90	22.00	22.90	22.90	22.90
x_4	14.20	14.20	14.20	14.50	14.20	14.20	14.20
x_5	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_6	1.62	1.62	1.62	1.62	1.62	1.62	1.62
x_7	7.97	7.97	7.97	7.97	7.97	7.97	7.97
x_8	22.90	22.90	22.90	22.90	22.90	22.90	22.90
x_9	22.00	22.00	22.00	22.00	22.00	22.00	22.00
x_{10}	1.62	1.62	1.62	1.62	1.62	1.62	1.62
W^*(lb$_f$)	5490.74	5490.74	5490.74	5469.14	5490.74	5490.74	5490.74
NSA	2,050	2,650	2,880	7,950	139	146	143
EET(s)	697	901	979	2,703	186	188	187
CV(%)	0	0	0	0.38	0	0	0

had the smallest *NSA* among existing studies. When comparing *EET*, the *EET* of this study is only 1.1~55.3% compared with existing studies.

3.2 25-bar truss

The discrete design of spatial 25-bar truss structure[13-15] shown in Fig. 3 is to determine the cross-sectional areas grouped into eight as follows: $x_1(A_1)$, $x_2(A_2\sim A_5)$, $x_3(A_6\sim A_9)$, $x_4(A_{10}\sim A_{11})$, $x_5(A_{12}\sim A_{13})$, $x_6(A_{14}\sim A_{17})$, $x_7(A_{18}\sim A_{21})$, and $x_8(A_{22}\sim A_{25})$. This truss structure is made of the material with ρ=0.1 lb/in^3 and E=10,000 ksi. The discrete design variables are selected from the set, S={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 ,1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4} in^2. For the sizing optimization problem, δ_{all} is 0.35 in, and σ_{all} is 40,000 psi. The loading condition is summarized in [13-15].

From Table 1, we see that the same optimal solution is calculated even though the initial values are changed. The results are summarized in Table 3, compared with existing studies. The first iteration (*iter*=1) in the design process gives the optimal solution. All studies except HAD, which provides the infeasible solution, yield the same results. Looking at the *NSA*, this study is the smallest number, followed by NMA with 250. All other existing studies have more than 1000. The *NSA* of this study is only 0.42~52.0 % of that of existing studies. On *EET*, NMA has the smallest value, followed by this study.

Table 3. Comparison of discrete designs for the 25-bar truss

x^* (in^2)	HPSO [4]	SA [3]	MBA [6]	aeDE [11]	SSA [10]	HHS [12]	EFA [20]	ACCS [13]	NMA [14]	HDA [15]	Present study		
											Case 1	Case2	Case3
x_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.3	0.3	0.3
x_3	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
x_4	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_5	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	1.9	2.1	2.1	2.1
x_6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
x_7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.5	0.5	0.5
x_8	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
W^*(lb$_f$)	484.85	484.85	484.85	484.85	484.85	484.85	484.85	484.85	484.85	482.83	484.85	484.85	484.85
NSA	25,000	7,900	2,150	1,678	5,050	5,000	1,300	2,560	250	7,450	104	120	130
EET()	9,063	2,864	779	608	1,831	1,813	471	928	91	2,701	252	258	262
CV(%)	0	0	0	0	0	0	0	0	0	0.6	0	0	0

3.3 72-bar truss

The discrete design of spatial 72-bar truss structure[12,13,15] shown in Fig. 4 is to determine the cross-sectional areas grouped into sixteen as follows: x_1(A_1~A_4), x_2(A_5~A_{12}), x_3(A_{13}~A_{16}), x_4(A_{17}~A_{18}), x_5(A_{19}~A_{22}), x_6(A_{23}~A_{30}), x_7(A_{31}~A_{34}), x_8(A_{35}~A_{36}), x_9(A_{37}~A_{40}), x_{10}(A_{41}~A_{48}), x_{11}(A_{49}~A_{52}), x_{12}(A_{53}~A_{54}), x_{13}(A_{55}~A_{58}), x_{14}(A_{59}~A_{66}), x_{15}(A_{67}~A_{70}), and x_{16}(A_{71}~A_{72}). This truss structure is made of the material with ρ=0.1 lb/in^3 and E=10,000 ksi, and the discrete design variables are selected from the set, S={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 ,1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2} in^2. δ_{all} and σ_{all} are given as 0.25 in and 25,000 psi, respectively. The loading condition is summarized in [12,13,15]. Table 4 shows the final results compared with existing studies. Although the HDA has the smallest weight, it is an infeasible design. Compared with existing studies, this study's *NSA* is only 0.62~9.68% and the *EET* is only 2.09~32.72%.

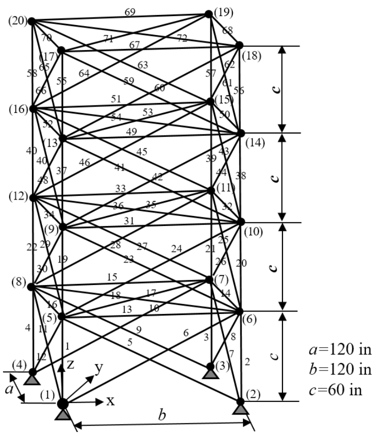

Fig. 4. 72-bar truss structure

4. Concluding remarks

Although the structural sizing optimization technique, including sensitivity analysis in continuous design space, is well established in theory, applying it to discrete designs is not general. Instead, most recent studies have used heuristic algorithms, which inevitably result in increased structural analyses. This research proposes a fast and economical design based on the optimal solution obtained from the structural sizing optimization techniques in continuous space and machine learning tools. The proposed design process, if the number of design variables is known, the numbers of training data and test data are determined, and a discrete optimal solution is calculated based on this, which is close to a deterministic way.

Table 4. Comparison of discrete designs for the 72-bar truss

	HPSO [4]	MBA [5]	IMBA [6]	IMCSS [9]	SSA [10]	HHS [12]	ACCS [13]	HDA [15]	Present study		
									Case 1	Case 2	Case 3
x_1	2.1	2.0	1.9	2.0	2.0	1.9	2.0	2.3	2.0	2.0	2.0
x_2	0.6	0.6	0.5	0.5	0.5	0.5	0.5	0.4	0.5	0.5	0.5
x_3	0.1	0.4	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_4	0.1	0.6	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_5	1.4	0.5	1.4	1.3	1.3	1.4	1.3	1.2	1.3	1.3	1.3
x_6	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_7	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_8	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_9	0.5	1.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{10}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{11}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_{12}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_{13}	0.2	1.9	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2
x_{14}	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
x_{15}	0.3	0.1	0.4	0.4	0.4	0.4	0.4	0.3	0.4	0.4	0.4
x_{16}	0.7	0.1	0.6	0.6	0.6	0.6	0.6	0.8	0.6	0.6	0.6
$W^*(lb_f)$	388.94	385.54	385.54	385.54	385.54	385.54	385.54	379.2	385.54	385.54	385.54
NSA	50,000	9,450	3,200	3,625	5,050	5,000	12,000	7,800	306	310	297
EET(s)	24,688	4,666	1,580	1,790	2,493	2,469	5,925	3,851	515	517	511
CV(%)	0	0	0	0	0	0	0	35.59	0	0	0

The main motivation of this research is to provide a discrete design while reducing the number of structural analyses as many as possible. This research is compared with existing studies through three examples and the following results are observed. The *NSA*s are only 0.38~14.60%, 0.42~52.0%, and 0.62~9.68% for the 10-bar, 25-bar, and 72-bar trusses, respectively. In comparison, the *EES* of existing studies is not possible to measure exactly and it represents only the estimated total calculation time obtained by multiplying the number of structural analyses times the one-analysis time based on the computer used in this study. However, the *EET* of this study is the measured time taken on the entire process from continuous optimization to discrete optimization. In the case of the 25-bar truss design, the *EET* of NMA is 34.73% smaller than that of this study. Excluding that, the *EET* of this study is only 1.10~55.30%, 3.14~54.77%, and 2.09~32.59% for the 10-bar, 25-bar, and 72-bar trusses, respectively. The discrete designs of three examples converge to the same optimum point regardless of the initial values, but this is not theoretically guaranteed. In future research, the suggested method will be applied to the shape optimization problems design of truss structures.

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