

테포시스템에서의 수송조건을 고려한 다품종발주모델에 관한 연구

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A Study for Multi-items Ordering Model with transportation on the Depot System

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요 약 본 논문은 전자상거래의 발전으로 인해 더욱 그 중요성이 부각되고 있는 중간창고(테포)에서의 발주시스템에 관한 연구이다. 제안 모델의 특징으로는 종래의 발주점 개념에서 발주범위라는 새로운 개념을 도입한 것이다. 제안 모델에서는 종래의 발주점 방식에서 고려하지 못 했던 다품종 발주와 발주시 적제효율을 고려한 수송조건 등을 포함하여 정식화하고 모델의 해법을 제안하고 있다. 또한 수치 예를 통하여 제안 모델의 유용성을 보여주고 있다.

Abstract In this paper, we propose a new ordering model for the mixed parts transportation problem with multi-items based on the depot system. Order scale are used as decision parameters instead of order point for ordering multi-items. Finally we test the model with simple example and show computational result that verifies the effectiveness of the model.

Key Words : Multi-items ordering system, Depot system, Order scale, Mixed parts transportation, Heuristics

1. Introduction

The automobile parts transportation problem from supplier to automobile maker has been one of the major issues on inventory and cost management in the automobile industry. In this paper, we propose a ordering model for multi-items based on the depot (distribution center) system. The depot system is being utilized to adjust the inventory and the timely supply of the parts demanded by automobile maker. Actually, it is need in the case of JIT supply for small lot and the distance from the supplier to automobile maker in long.

There has been several models about the multi-items ordering system. Shu [1] proposed a ordering model for multi-items firstly. He determined the optimal ordering cycle with respect to the rate of demands for two items. Nocturne [2] formulated a model for joint replenishment of two items with his proposed original logic. Doll and Whybark [3]

determined near optimal frequencies of production and the associated cycle time for production schedules.

They determined optimal ordering cycle as decision parameters without considering transportation condition.

Tersine and Barman [4] constructed a model considering transportation condition (freight rate) for determining ordering size in a deterministic EOQ system. Buffa and Munn [5] proposed an algorithm that could formulate an inbound consolidation strategy. The model is available to the case of replenishing multi-items with some groups and the cost of transportation and inventory being affected by the strategy.

But these models are not considered the transportation condition and multi-items together. Therefore, we propose a new ordering model on the fixed order method with order scale in order to overcome the above mentioned shortcomings. The proposed model is considered the multi-items and transportation condition.

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2. MODEL DESCRIPTION

2.1 Concept

In the fixed order point method, an order is placed when the inventory on hand reaches a predetermined inventory level, to satisfy demand during the order cycle. The economic order quantity will be ordered whenever demand drops the inventory level to the reorder point. This method is based on the following assumptions; (1) A continues, constant and known rate of demand. (2) A constant and known replenishment or lead time. (3) Only one item in inventory. (4) An infinite planning horizon. etc.

It is difficult to consider the impact of multi-items and transportation condition in the typical fixed order method. For this reason, the model proposed in this paper use order scale as decision parameters instead of order point. Therefore, the order policy are changed as follows. We show the order policy of the proposed model for two items (a, b).

Case 1: When the inventory level of a and b are within the order scale of a and b. If the sum of transportation order quantities (a, b) is in the limits of the available transportation quantity, order a and b, else do not order a and b.

Case 2: greater than the upper order scale of a . greater than the order scale of b. Do not order a and b.

Case 3: greater than the order scale of a. within the order scale of b. Do not order a. If the transportation order quantity (b) is in the limits of the available transportation quantity, order b, else do not order b.

Case 4: greater than the order scale of a. less than the order scale of b. Do not order a. Order b.

Case 5: within the order scale of a. less than the order scale of b. If the transportation order quantity (a) is in the limits of the available transportation quantity, order a, else do not order a. Order b.

Case 6: less than the order scale of a. less than the order scale of b. Order a and b.

Figure 1 shows the behavior of inventory level on proposed model. In Figure 1, the transportation order for the item a and b are placed on the order scale predetermined respectively. The transportation quantity of the item a and b are Q_a and Q_b respectively.

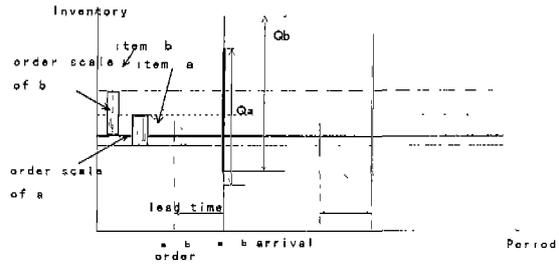


Figure 1. Behavior of inventory level on the depot in the proposed model

The decision factor of proposed model is as follows.

- (1) determination of the order scales,
- (2) determination of transportation order quantity.

The model is formulated for deciding maximum point of order scale and transportation order quantity in such a way that total cost is minimized. The minimum point of order scale is determined by adapting the order point in the typical fixed order method.

2.2 Assumption

The material and information flows in the proposed model is shown in Figure 2. We assume that transportation lead time is L_1 and demands are supplied to assembly line from the depot timely. Supplier produce M types of items and let $i \in \{1, 2, \dots, M\}$ be index of items. Let $i \in \{1, 2, \dots, T\}$ be an index of the time periods with the condition that the planning horizon starts at the beginning of period 1 and the end of period T . The set of time period is H .

The assumption used in this paper are as follows.

- (1) Demands for depot are deterministic process and suggested by assembly line.
- (2) Inventory quantities in supplier's inventory

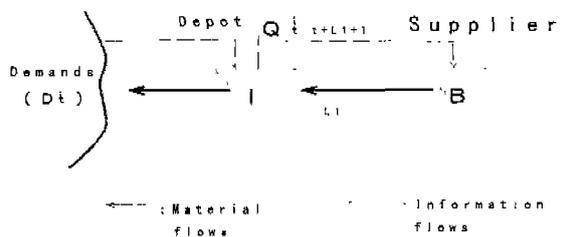


Figure 2. Material and information flows in the proposed model

point are always sufficient.

(3) During the planning horizon, inventory cost and transportation cost are constant.

(4) A minimum and maximum shipping quantity on the transportation are predetermined.

(5) Transportation for one period occurs once or not.

2.3 Notation

k : time periods on transportation lead time

($k = 1, 2, \dots, L1$)

D_t^i : demands for item i in period t

S^i : safety inventory level for item i

C_0^i : holding cost in the depot for item i

C_1 : transportation cost

Z^i : container size for item i

F_{\min}, F_{\max} : minimum and maximum available transportation size considering the mixed parts transportation and freight rate

W^i : value of minimum of order scale for item i (this value is equal to order point in order point method)

I_t^i : inventory in the depot for item i in period t

Y_t : variable of transportation in period t

$Q_{t:L1+1}^i$: transportation quantity ordered in period t for item i , this quantity is transported at the beginning of period $t+L1+1$

V^i : value of maximum in order scale for item i (decision variable)

G^i : set of transportation quantity for item i ,

$$G^i = \left\{ g^i | g^i = \left\lceil \left(\sum_{t=1}^T D_t^i \right) / t \right\rceil, t \in H \right\}$$

$\lceil X \rceil$: minimum integer larger than X

2.4 Formulation

(1) The inventory quantity on the depot:

$$I_t^i = I_{t-1}^i + Y_{t-L1-1} \times Q_{t-L1-1:t}^i - D_t^i \quad (t = 1, 2, \dots, T; i = 1, 2, \dots, M) \quad (1)$$

I_0^i : initial inventory level for items i on the depot

$Y_{-L1} \times Q_{-L1:1}^i, \dots, Y_0 \times Q_{0:L1}^i$: preordered transportation quantity

(2) The transportation order quantity:

$$Q_{t:t+L1+1}^i = \begin{cases} 0 & \text{if } I_t^i + \sum_{k=1}^{L1} Y_{t-k} \times Q_{t-k:t+L1+1-k}^i > V^i \\ g^i & \text{if } I_t^i + \sum_{k=1}^{L1} Y_{t-k} \times Q_{t-k:t+L1+1-k}^i \leq V^i \end{cases} \quad (t = 1, 2, \dots, T; i = 1, 2, \dots, M) \quad (2)$$

(3) The criterion of transportation:

$$Y_t = \begin{cases} 1 & \text{if } F_{\min} \leq \sum_{i=1}^M Z^i \times Q_{t:t+L1+1}^i \leq F_{\max} \quad \text{or} \\ 0 & \text{otherwise} \end{cases}$$

$$\exists i \in (1, 2, \dots, M), I_t^i \sum_{k=1}^{L1} Y_{t-k} \times Q_{t-k:t+L1+1-k}^i < W^i \quad (t = 1, 2, \dots, T) \quad (3)$$

(4) The condition for next planning horizon:

$$\sum_{t=1}^T Y_t \times Q_{t:t+L1+1}^i \geq \sum_{t=1}^T D_t^i \quad (i = 1, 2, \dots, M) \quad (4)$$

(5) The condition of safety inventory quantity:

$$I_t^i \geq S^i \quad (t = 1, 2, \dots, T; i = 1, 2, \dots, M) \quad (5)$$

(6) The set of transportation order quantity in our model:

$$G^i = \left\{ g^i | g^i = \left\lceil \left(\sum_{t=1}^T D_t^i \right) / t \right\rceil, g^i \leq F_{\max} / Z^i, t \in H \right\} \quad (i = 1, 2, \dots, M) \quad (6)$$

(7) The range of maximum of order scale:

$$V_{\min}^i \leq V^i \leq V_{\max}^i \quad (i = 1, 2, \dots, M) \quad (7)$$

(8) The minimum of maximum of order scale:

$$V_{\min}^i = \left\lceil L1 \times \bar{D}^i + \alpha \times L1 \times \delta_D^i \right\rceil \quad (i = 1, 2, \dots, M) \quad (8)$$

(9) The maximum of maximum of order scale:

$$V_{\max}^i = g_{\max}^i + S^i \quad (i = 1, 2, \dots, M) \quad (9)$$

(10) The minimum of transportation order quantity:

$$g_{\min}^i \leftarrow \min G^i \quad (i = 1, 2, \dots, M) \quad (10)$$

(11) The maximum of transportation order quantity:

$$g_{\max}^i \leftarrow \max \mathbf{G}^i \quad (i = 1, 2, \dots, M) \quad (11)$$

where,

- \bar{D}^i : average demands of item i
- $L1$: transportation lead time
- α : safety coefficient (in this model, $\alpha = 1.65$)
- δ_D^i : standard deviation of demands of item i

From the above formulation, we define an optimization equation as follows. This equation means the minimization of the sum of holding cost in the depot, transportation cost between depot and supplier.

Minimize

$$TC_T = \sum_{t=1}^T \sum_{i=1}^M C_0^i \times I_t^i + C_1 \times \sum_{t=1}^T Y_t \quad (12)$$

s. t. (1)-(7)

3. COMPUTATIONAL PROCEDURE

The computational procedure to determine the transportation order quantities and order scales are as follows.

Step 1

- 1) Input the initial data:
 $S^i, Z^i, I_0^i, C_1, L1, C_0^i, T, M, F_{\min}, F_{\max}, D_t^i$,
 where, $i = 1, 2, \dots, M; t = 1, 2, \dots, T$.

2) Set the initial value of parameters:

$$TC_{opt} \leftarrow \infty, \Delta V \leftarrow 1, t \leftarrow 1, TT \leftarrow T, V_{\min}^i, V_{\max}^i, g_{\min}^i, g_{\max}^i$$

Step 2

Calculate equations (1), (2) for item i .

If $I_t^i + \sum_{k=1}^{L1} Y_{t-k} \times Q_{t-k:t+L1+1-k}^i > V_{\min}^i$ then set

$$Q_{t:t+L1+1}^i \leftarrow 0, Q_{t:t+L1+1}^i \leftarrow g_{\min}^i. \text{ And set}$$

$$SQ \leftarrow \sum_{i=1}^M Z^i \times Q_{t:t+L1+1}^i \text{ go to Step 3.}$$

Step 3

For item i . If $I_t^i > S^i$, then go to Step 8, else go to Step 4.

Step 4

Calculate equation (3). If $F_{\min} \leq SQ \leq F_{\max}$, or if

$$I_t^i + \sum_{k=1}^{L1} Y_{t-k} \times Q_{t-k:t+L1+1-k}^i > W^i \text{ for item } i, \text{ set}$$

$Y_t \leftarrow 1$, then go to Step 5, else set $Y_t \leftarrow 0$ go to Step 5.

Step 5

Calculate equation (6) at period t .

- 1) If $t < T$, then set $t \leftarrow t+1$ go to Step 2.
- 2) If $t = T$, then go to Step 6.

Step 6

Judge equation (4). For item i , if $\sum_{i=1}^T Y_t \times Q_{t:t+L1+1}^i < \sum_{i=1}^T D_t^i$, then go to Step 8, else go to Step 7.

Step 7

- 1) If $TC_T < TC_{opt}$, then set $TC_{opt} \leftarrow TC_T, V_{opt}^i \leftarrow V_{\min}^i, g_{opt}^i \leftarrow g_{\min}^i$, for item i , go to Step 8
- 2) If $TC_T \geq TC_{opt}$, then go to step 8.

Step 8

For item i ,

- 1) if $g_{\min}^i < g_{\max}^i$, then set $TTTT \leftarrow TT-1, g_{\min}^i \leftarrow$

$$\left[\left(\sum_{t=1}^T D_t^i \right) / TTT \right], t \leftarrow 1, \text{ go to Step 2,}$$

- 2) if $g_{\min}^i \geq g_{\max}^i$, then set $t \leftarrow 1$ go to Step 9.

Step 9

For item i ,

- 1) if $V_{\min}^i < V_{\max}^i$, then set $V_{\min}^i \leftarrow V_{\min}^i + \Delta V$, go to Step 2,
- 2) if $V_{\min}^i \geq V_{\max}^i$, then go to Step 10.

Step 10

If $TC_{opt} = \infty$, print "infeasible" and end, else print $TC_{opt}, V_{opt}^i, g_{opt}^i$ for item i , and end.

4. NUMERICAL EXAMPLE

We consider numerical example in order to demonstrate the effectiveness of the model developed.

We assume that there are 3 types of items and the planning horizon starts at the beginning of period 1 and finishes at the end of period T=10. And transportation lead time is 1 period. Following data are used in this paper.

4.1 Input Data

(1) demands (container)

$t =$	1	2	3	4	5	6	7	8	9	10
D_t^1	9	11	11	10	9	8	9	11	9	11
D_t^2	18	19	22	24	24	20	21	18	20	22
D_t^3	28	32	32	30	31	30	33	28	30	30

(2) size for each items (m^3)

$Z^1=3 \quad Z^2=2 \quad Z^3=1$

(3) minimum and maximum available transportation size (m^3)

$F_{min} = 180 \quad F_{max} = 220$

(4) holding cost and transportation cost (\$)

$C_0^1=1 \quad C_0^2=2 \quad C_0^3=1 \quad C_1=100$

(5) initial inventory level, safety inventory level (container)

$I_0^1=10 \quad I_0^2=27 \quad I_0^3=39 \quad S^i=1 \quad (i = 1, 2, 3)$

(6) preordered transportation quantity (container)

$Q_{-1:1}^i=0 \quad (i = 1, 2, 3) \quad Q_{0:2}^1=23 \quad Q_{0:2}^2=40$

$Q_{0:2}^3=61 \quad Y_{-1}=0 \quad Y_0=1$

4.2 Computational Result

Through the computational procedures with above input data, we obtained the Table 1, 2, 3 and Figure 3. Table 1 shows the value of computational results. The transportation on the planning horizon occurs 5 times. Total cost is 1,185. The transportation orders (20 of item 1, 42 of item 2 and 61 of item 3) are occurred at end of period of 2, 4, 6, 8, 10 (Table 2). Table 3 shows the inventory level in the depot.

Table 1. Value of computational results

Item $i =$	1	2	3
Transportation order quantity	20	42	61
Occurrence of order	5	5	5
Order scale	13-15	25-30	34-40
Total inventory quantity	88	179	239

Table 2. Transportation order quantities

$t =$	1	2	3	4	5	6	7	8	9	10
$Q_{t:t+L+1}^1$	0	20	0	20	0	20	0	20	0	20
$Q_{t:t+L+1}^2$	0	42	0	42	0	42	0	42	0	42
$Q_{t:t+L+1}^3$	0	61	0	61	0	61	0	61	0	61

Table 3. Inventory level on the depot

$t =$	1	2	3	4	5	6	7	8	9	10
I_t^1	1	13	2	12	3	15	6	15	6	15
I_t^2	9	30	10	28	4	26	5	29	9	29
I_t^3	11	40	8	39	8	39	6	39	9	40

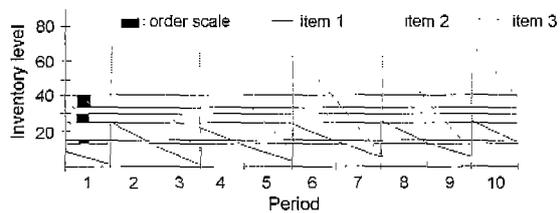


Figure 3. Behavior of inventory level on the depot.

Figure 3 illustrates the behavior of inventory level in the depot. The inventory level at period of 1 is less than the order scale, but order was not placed on this period because there were preordered transportation quantity. The order was placed on the period 2 because the inventory levels on hand were within the order scale.

From this example, we can conclude that

- (1) the mixed parts transportation for multi-items occur all period (see Table 2)
- (2) the occurrence of order is minimized by the proposed model under total minimum cost.

5. CONCLUSION

In this paper, we proposed the ordering model for multi-items in consideration of the transportation condition. The contributions of this paper are as follows.

- (1) A new ordering model for multi-items with order scale proposed.
- (2) The proposed model considered the mixed

parts transportation.

(3) A numerical experiment to verify the effectiveness of proposed model was demonstrated.

We used to the personal computer for calculating our numerical example, it is necessary to develop our model to calculate simply and fastly for the further study.

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