한국산학기술학회논문지
Vol. 7, No. 2, pp. 251-256, 2006

AN ORDERING MODEL TO DETERMINE PRODUCTION QUANTITY IN JUST-IN-TIME PRODUCTION SYSTEM

Beum-Jun Ahn[1*]

JIT 생산시스템에서의 발주량 결정을 위한 모델 설계

안범준[1*]

Abstract In this paper we consider multi-stage, multi-product production, inventory systems which have assembly-tree-structure. We propose a new mathematical model for pull type ordering systems based on JIT manufacturing systems. To apply the model to an actual automobile parts manufacturer, the objective of proposed model is to minimize the sum of inventory and setup costs. Finally, a numerical example and computational results are given to illustrate the proposed model.

요 약 본 논문은 다단계, 다품종 조립라인을 고려한 모델이며 JIT 생산시스템에서의 최적의 발주량 계산을 목적으로 하고 있다. 제안 모델의 목적함수는 재고비와 setup 비의 합이 최소화되도록 설계되어 있다. 이는 대부분의 자동차 부품회사의 경우 setup 공정을 포함하고 있기 때문이다. 마지막으로 수치실험을 통해 제안 모델의 유효성을 보여 주고 있다.

Key Words : Multi-items ordering model, inventory and setup costs, JIT production system

1. INTRODUCTION

The decision process in production planning involves many complex problems and the decision maker is confronted with conflicting factors. For example, there is the minimum safety inventory problem. This field has received much attention in production control literature. There are many different models and solution methods have been published and applied([1], [2], [3]).

There has been increased interest in the Japanese JIT technique with Kanbans by production managers. JIT production system is the system by which only the necessary products and produced in the necessary quantities at the necessary time([4], [5], [6]). The objective is to keep a constant minimum inventory level.

In JIT production system research, Bitran and Chang[2] first proposed a model to determine the number of Kanbans with mathematical programming. In the model, they provide a nonlinear programming originally. Next, they transformed the resulting model to a mixed integer programming model and then to a linear programming model. Their model is built for single-product multi-stage production process on single-card Kanban system. Bard and Golany[1] proposed another mathematical programming model to determine the number of Kanban involving the concept of setup, lead time for the multi-product production system. On the other hand, Moeeni and Chang[3] proposed a simple heuristic model for computing the number of Kanbans in the system which has a multi-stage, uncapacitated, assembly-tree-structure, with every stage producing only one item at a time.

2. MODEL DESCRIPTION

In this paper, we consider a multi-stage production process with dual Kanban and that the final assembly process produces multiple products. And the model

[1]Department of Industrial Information & Systems Engineering, Sangmyung University
*Corresponding Author:Beum-Jun Ahn(bjahn@smu.ac.kr)

applied to an automobile parts manufacturer consists of N stages. Let $n \in \{1, 2, ..., N\}$ be an index of the stages with the condition that $n=1$ stands for the final stages. Each stage $n \in \{1, 2, ..., N\}$ means a production process and includes an immediately succeeding inventory point and an on-hand inventory point for the immediately succeeding stage. Let $t \in \{0, 1, ..., T\}$ be an index of the time periods with the condition that the planning horizon starts at the beginning of period 1 and finishes at the end of period T.

We assume the applied model satisfies the following conditions.

(1) Demands for final products in each period is suggested by customers.
(2) Each stage produces M types of items and let $i \in \{1, 2, ..., M\}$ be an index of items.
(3) The quantity of production and transportation ordered for each stage is calculated at the end of the preceding period.
(4) The processing time for each item at each stage is known and fixed in the planning horizon.
(5) One full container of item i at each stage is exactly required to make one full container of item i at the immediately succeeding stage.

Therefore, parameters and variables for production, inventory and transportation quantity in the model stand for the number of Kanbans.

2.1 Parameters

$s(n)$:immediately succeeding stage of stage n;
$(n=2, 3, ..., N)$

J_t^n:operation time at production process of stage n in period t;
$J_t^n \geq 0 \quad (n=1, 2, ..., N; t=1, 2, ..., T)$

$a^{n(i)}$:processing time required to make one full container of item i at production process of stage n;
$(i=1, 2, ..., M; n=1, 2, ..., N)$

$D_t^{(i)}$:demand for the final product i in period t;
$D_t^{(i)} \in \{0, 1, 2, ...\}$
$(i=1, 2, ..., M; t=1, 2, ..., T)$

$B_0^{n(i)}$:initial on-hand inventory quantity of final product i available to customers($n=1$) and item of stage n for production process of stage $s(n)$ $(n=2, 3, ..., N)$
$B_0^{n(i)} \in \{0, 1, 2, ...\} \quad (i=1, 2, ..., M)$

$I_0^{n(i)}$:initial inventory quantity of final product i ($n=1$) and item i fabricated by stage n $(n=2, 3, ..., N)$;

$C_0^{n(i)}$:inventory cost for one full container of item i fabricated by stage n;
$(i=1, 2, ..., M; n=1, 2, ..., N)$

$K^{n(i)}$:setup cost for item i fabricated by stage n;
$(i=1, 2, ..., M; n=1, 2, ..., N)$

2.2 Variables

$B_t^{n(i)}$:on-hand inventory quantity of final product i available to customers($n=1$) and item i of stage n($n=2, 3, ...N$) at the end of period t;
$(i=1, 2, ..., M; t=1, 2, ..., T)$.

$I_t^{n(i)}$:inventory quantity of final product i($n=1$) and item i fabricated by stage n($n=2, 3, ...N$) at the end of period t;
$(i=1, 2, ..., M; t=1, 2, ..., T)$.

$P_{t-1:t}^{n(i)}$:actual production quantity in period t which is ordered by production-ordering Kanban of final product i for final stage $n=1$ and item i for stage n($n=2, 3, ...N$) at the end of period
$t-1$; $(i=1, 2, ..., M; t=1, 2, ..., T)$.

$d_{t-1:t}^{n(i)}$:actual withdrawal quantity in period t which is ordered by withdrawal Kanban of final product i for final stage $n=1$ and item i for stage n($n=2, 3, ...N$) at the end of period $t-1$; $(i=1, 2, ..., M; t=1, 2, ..., T)$.

$Y_{t-1:t}^{n(i)}$:variable to represent setup at production process of item i for stage n at period t;
$(i=1, 2, ..., M; n=1, 2, ..., M; t=1, 2, ..., T)$.

Variables for the number of production-ordering and withdrawal Kanban which are presented to each stage by managers at the beginning of planning horizon are as follows.

$U_0^{n(i)}$:number of production-ordering Kanban of final product i for final stage $n=1$ and item i for stage $n(n=2,3,\ldots N)$; $(i=1,2,\ldots,M)$.

$V_0^{n(i)}$:number of withdrawal Kanban of final product i for final stage $n=1$ and item i for stage n $(n=2,3,\ldots N)$; $(i=1,2,\ldots,M)$.

Variables for the number of production-ordering and withdrawal Kanban which are calculated and presented as order to each stage at the period t $(t=1,2,\ldots,T)$ are as follows.

$U_t^{n(i)}$:number of production-ordering Kanban of final product i for final stage $n=1$ and item i for stage $n(n=2,3,\ldots N)$ at the end of period t; $(i=1,2,\ldots,M; t=1,2,\ldots,T)$.

$V_t^{n(i)}$:number of withdrawal Kanban of final product i for final stage $n=1$ and item i for stage n $(n=2,3,\ldots N)$ at the end of period t; $(i=1,2,\ldots,M; t=1,2,\ldots,T)$.

We denote these notations by column vectors as follows.

$$a^n=[a^{n(i)}],\quad D_t=[D_t^{n(i)}],\quad B_0^n=[B_0^{n(i)}],$$
$$I_0^n=[I_0^{n(i)}],\quad B_t^n=[B_t^{n(i)}],\quad I_t^n=[I_t^{n(i)}],$$
$$U_0^n=[U_0^{n(i)}],\quad V_0^n=[V_0^{n(i)}],\quad U_t^n=[U_t^{n(i)}],$$
$$V_t^n=[V_t^{n(i)}],\quad P_{t-1:t}^n=[P_{t-1:t}^{n(i)}],$$
$$d_{t-1:t}^n=[d_{t-1:t}^{n(i)}],\quad Y_{t-1:t}^n=[Y_{t-1:t}^{n(i)}]$$
$$(i=1,2,\ldots,M;\ n=1,2,\ldots,N;\ t=1,2,\ldots,T)$$

Fig. 1 shows a conceptual diagram of the model for multi-stage capacitated assembly-tree-structured production system producing M types of item at each stage.

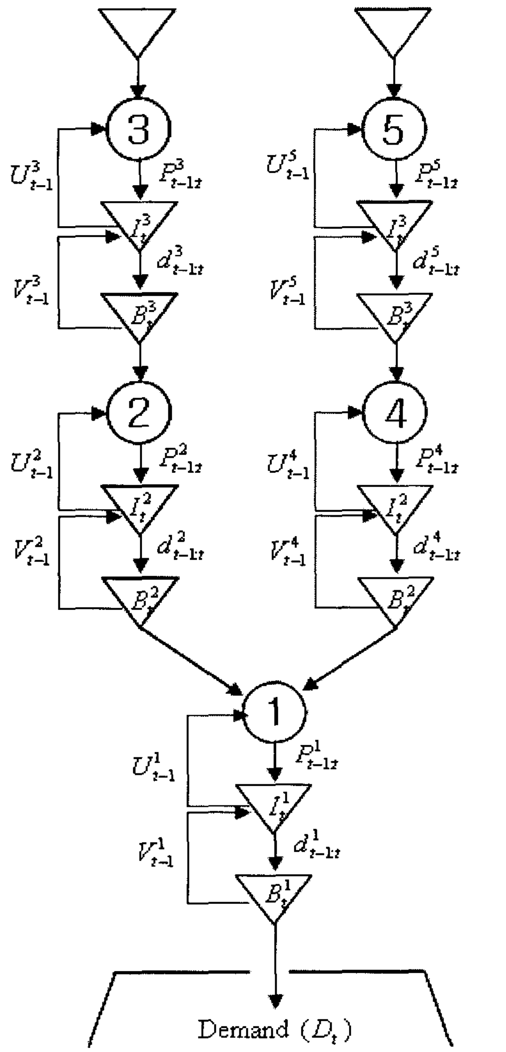

Fig. 1 A conceptual diagram of multi-stage production system($n=5$)

2.3 Formulations of pull type ordering model

Using the notation defined above, we formulate a mathematical model for pull type ordering systems based on JIT manufacturing systems as follows.

$$B_t^1=B_{t-1}^1+d_{t-1:t}^1-D_t \qquad (1)$$
$$(t=1,2,\ldots,T)$$
$$B_t^n=B_{t-1}^n+d_{t-1:t}^n-P_{t-1:t}^{s(n)} \qquad (2)$$
$$(n=2,3,\ldots,N;\ t=1,2,\ldots,T)$$
$$I_t^n=I_{t-1}^n+P_{t-1:t}^n-d_{t-1:t}^n \qquad (3)$$
$$(t=1,2,\ldots,T)$$

$$V_t^1 = V_{t-1}^1 - d_{t-1:t}^1 + D_t \quad (4)$$
$$(t = 1, 2, \ldots, T)$$
$$V_t^n = V_{t-1}^n - d_{t-1:t}^n + P_{t-1:t}^{s(n)} \quad (5)$$
$$(n = 2, 3, \ldots, N;\ t = 1, 2, \ldots, T)$$
$$U_t^n = U_{t-1}^n - P_{t-1:t}^n + d_{t-1:t}^n \quad (6)$$
$$(n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$
$$d_{t-1:t}^n \le V_{t-1}^n \quad (7)$$
$$(n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$
$$P_{t-1:t}^n \le U_{t-1}^n \quad (8)$$
$$(n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$
$$Y_{t-1:t}^n = \begin{cases} 0, & \text{if } P_{t-1:t}^{n(i)} = 0 \\ 1, & \text{if } P_{t-1:t}^{n(i)} > 0 \end{cases} \quad (9)$$
$$(i = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$
$$\sum_{i=1}^{M} a_{t-1:t}^{n(i)} P_{t-1:t}^{n(i)} \le J_t^n \quad (10)$$
$$(n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$

$B_t^n,\ I_t^n,\ U_t^n,\ V_t^n,\ P_{t-1:t}^n,\ d_{t-1:t}^n,\ U_0^n,\ V_0^n$: nonnegative integer

$$(n = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T)$$

Under the above mentioned constraints(1)~(11), we consider the following optimize program which is minimized the objective function(12). The term of the objective function(12) indicates the sum of inventory costs which are represented by the number of initial Kanban. The second term indicates the sum of setup costs:

Minimize

$$F = \sum_{n=1}^{N} \sum_{i=1}^{M} C^{n(i)} (V_0^{n(i)} + B_0^{n(i)} + U_0^{n(i)} + I_0^{n(i)}) + \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{i=1}^{M} K^{n(i)} Y_{t-1:t}^{n(i)} \quad (12)$$

s.t. (1)~(11)

The model we formulated above has the following characteristics;

(1) Dual card Kanban is adopted.

(2) The concept of setup is considered.

(3) Multi-product production system is adopted.

3. APPLICATION TO AUTOMOBILE PARTS MANUFACTURER

In order to demonstrate the effectiveness of the model developed in the above section, we apply the model to an actual automobile parts manufacturer and make a numerical experiment using the mathematical programming package. The system consists of 5 stages and its diagram is the same as Fig. 1. Each production process corresponding to Fig. 1 is as follows;

(1) assembly process : $n=1$

(2) press process 1 : $n=2$

(3) press process 2 : $n=3$

(4) bending process : $n=4$

(5) cutting process : $n=5$

Each stage produces 3 types of the item and press processes (stage 2 and 3) require the setup.

3.1 Numerical experiments

We assume that the planning horizon starts at the beginning of period 1 and finishes at the end of period T=5 and each stage produces 3 types of item. Table 1, 2 and 3 show initial inventory quantity, demand for the final products and inventory cost respectively.

Table 1. Initial inventory quantity

	i=1	i=2	i=3		i=1	i=2	i=3
$B_0^{1(i)}$	5	4	1	$I_0^{1(i)}$	5	4	1
$B_0^{2(i)}$	6	5	2	$I_0^{2(i)}$	6	5	2
$B_0^{3(i)}$	6	5	2	$I_0^{3(i)}$	6	5	2
$B_0^{4(i)}$	6	5	2	$I_0^{4(i)}$	6	5	2
$B_0^{5(i)}$	6	5	2	$I_0^{5(i)}$	6	5	2

Table 2. Demand

	i=1	i=2	i=3
$D_0^{1(i)}$	20	15	5
$D_0^{2(i)}$	30	25	5
$D_0^{3(i)}$	30	25	5
$D_0^{4(i)}$	30	25	5
$D_0^{5(i)}$	20	15	5

Table 3. Inventory cost

	i=1 i=2 i=3
$C^{1(i)}$	5 6 7
$C^{2(i)}$	4 5 6
$C^{3(i)}$	3 4 5
$C^{4(i)}$	2 3 4
$C^{5(i)}$	1 2 3

3.2 Computational results

By computing for the above mentioned input data, we obtained the number of initial Kanban(Table 4) and quantities of actual production and production ordering (Table 5). Table 6 shows the behaviour of inventory quantity of stage 1.

Table 4. Number of initial Kanbans

	i=1 i=2 i=3		i=1 i=2 i=3
$V_0^{1(i)}$	27 25 4	$U_0^{1(i)}$	30 20 4
$V_0^{2(i)}$	29 17 3	$U_0^{1(i)}$	19 15 3
$V_0^{3(i)}$	28 17 3	$U_0^{1(i)}$	24 13 3
$V_0^{4(i)}$	29 17 3	$U_0^{1(i)}$	20 15 3
$V_0^{5(i)}$	20 15 3	$U_0^{1(i)}$	20 15 2

Table 5. Quantity of actual production and production ordering(assembly process: stage 1)

	i=1 i=2 i=3		i=1 i=2 i=3
$P_{0:1}^{1(i)}$	26 20 4	$U_0^{1(i)}$	30 20 4
$P_{1:2}^{1(i)}$	24 22 4	$U_1^{1(i)}$	19 15 3
$P_{2:3}^{1(i)}$	27 18 5	$U_2^{1(i)}$	24 13 3
$P_{3:4}^{1(i)}$	23 22 5	$U_3^{1(i)}$	20 15 3
$P_{4:5}^{1(i)}$	35 0 5	$U_4^{1(i)}$	20 15 2

Table 6. Behaviour of inventory quantity at the end period(assembly process: stage 1)

	i=1 i=2 i=3		i=1 i=2 i=3
$B_0^{1(i)}$	5 4 1	$I_0^{1(i)}$	5 4 1
$B_1^{1(i)}$	5 11 0	$I_1^{1(i)}$	11 2 1
$B_2^{1(i)}$	2 4 0	$I_2^{1(i)}$	8 6 0
$B_3^{1(i)}$	2 1 0	$I_3^{1(i)}$	5 2 0
$B_4^{1(i)}$	0 0 0	$I_4^{1(i)}$	0 0 0
$B_5^{1(i)}$	0 0 0	$I_5^{1(i)}$	15 0 0

From these tables, we obtain the following.

(1) If we present the number of initial Kanban such as Table 4 to each stage, this manufacturing system will be operated well.

(2) The actual quantities of production are not more than the number of production-ordering Kanban presented as order(Table 5).

4. CONCLUSIONS

In this paper we developed an optimization model for the multi-stage, multi-product production system with dual Kanbans. In order to clarify the effectiveness of the proposed model, we applied the model to an actual automobile parts manufacturer and carried out the numerical calculation.

As results, we obtained the following.

(1) By applying to automobile parts manufacturer, we knew that the proposed model gradually minimize the inventory level(Table 6).

(2) And we assist managers to determine the number of circulating Kanbans at each stage.

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Beum-Jun Ahn [Life member]

He received his B.S. degree in Industrial Engineering from Korea University, his degrees M.S. and Ph.D in Economics from Hiroshima National University respectively. He is currently an associate professor at Sangmyung University.

<Research interest>
Production system, Logistics, Quality Control