지반 크리프 거동의 미시학적 모델링

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Microscopic Modeling of Creep Behavior for Soils

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요 약 미시학적 비배수 크리프 현상의 누적 변형은 점성토 시공지역 지반의 전반 파괴를 야기 할 수 있다. 본 연구에서는 점성토의 비배수 크리프 거동을 예측하기 위하여 Perzyna의 일반 점성이론을 소성론의 개념에서 간략화하고 수정 Cam clay 모델 및 데미지 이론을 포함하는 하나의 시간의존적 구성방정식을 유도하였다. 유도된 구성방정식을 활용하여 예측한 크리프 거동은 비배수 크리프 파괴를 포함하는 크리프 실험결과와 잘 부합하였다.

Abstract The accumulated deformation due to the undrained creep causes the general stability problem for the overall soil mass. In this study, the time-dependent constitutive equation, into which a damage law, modified cam clay model, and Perzyna's generalized viscous theory were incorporated, was derived microscopically. The model prediction agreed well with the experimental result including the case of the undrained creep rupture.

Key Words : Microscopic, Constitutive Model, Creep, Damage, Elastic, Plastic

1. Introduction

An important time-dependent phenomenon is the creep in stability analysis for soil mass. The undrained creep rupture significantly affects the overall stability and should be taken into consideration[1, 2, 3, 4]. Most mathematical creep studies have regarded soils as very homogeneous throughout the entire soil mass; however, microscopically, soils in practice can be partly damaged during deformation[2, 3, 6, 9].

In this study, a combined elastic-plastic-viscous constitutive relation was developed. The generalized Hooke's law and the modified Cam Clay model were respectively used for the elastic and plastic parts.

The generalized viscous theory by Perzyna was simplified and the damage law by Vyalov(1986) was incorporated for obvious mathematical formulation in an elastic-plastic-viscous combination scheme[5, 6, 7, 8. 9]. The physical formulation of the combined model was performed from the point of view that fewer parameters better be employed. The model predictions were compared with the experimental results.

2. Development of Model

2.1 Mathematical Formulation

The combination and mathematical formulation of elastic, plastic, and viscous constitutive equations is as Eq. (1)[7]. The superscripts e, p, vp denote elastic, time-independent plastic, and time-dependent viscoplastic parts, respectively.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p} + \dot{\varepsilon}_{ij}^{vp} \tag{1}$$

The elastic strain rate is obtained using the generalized Hooke's law. The time-independent plastic strain rate is represented by the modified Cam Clay loading function(Eqs. 2 and 3).

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$$\dot{\varepsilon}_{ij}{}^{p} = \langle L \rangle \frac{\partial f}{\partial \sigma_{ij}} \qquad (2)$$

$$f(p, \ q, \ p_c(\varepsilon_v^p)) = M^2 p^2 - M^2 p_c p + q^2 \qquad (3)$$

where p and q respectively denote the mean effective stress and the deviatoric stress. Fig. 1 shows the yield surface in p-q space, where pc is the strain hardening parameter and the apex of the yield surface in p axis, with the isotropic hardening rule. The case of $p \geq p_c/2$ represents the strain hardening or the critical state, on the other hand, the case of $p < p_c/2$ represents the strain softening, that may produce tremendous error in numerical analysis. To overcome the numerical problem, the yield surface was used as the perfectly plastic line for strain softening region.
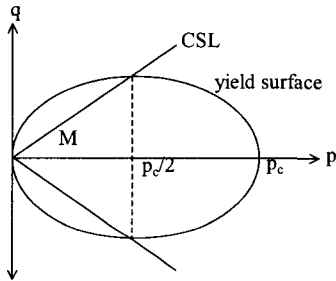
q

Fig. 1. Modified Cam Clay model

The soil damage is represented by the single damage variable ω, in the form of Eq. (4).

$$\omega = 1 - \frac{(1 - \omega_o)}{(1 + t)^{A \bar{\tau}}}$$

$$1 - \omega = (1 - \omega_o)(1 + t)^{-A \bar{\tau}} \qquad (4)$$

where ω is the degree of damage at any time t, ω_o is the initial structural damage, A is constant, and $\bar{\tau}$ is a dimensionless stress function representing the magnitude of an applied deviatoric stress. The t is a dimensionless quantity equal to the value of period of deformation t divided by t''', where t''' is a parameter measured in units of time, and may be taken to be equal to one. The quantities $1-\omega_o$ and $1-\omega$ define the undamaged areas of soil at the initial condition and at any time t, respectively. The

ω_f indicates the degree of damage at the moment of failure $(t=t_f)$. The $\bar{\tau}$ is assumed as in Eq. (5).

$$\omega = 1 - \frac{(1 - \omega_o)}{(1 + t)^{\frac{A\tau}{\tau_o - \tau}}} \qquad (5)$$

where τ_o is the hypothetical instantaneous strength of the soil, whose value is determined from the creep test; however, τ_o is assumed to be undrained shear strength so that it can be practically determined from the time-independent triaxial test. The τ is the applied deviatoric stress. Eq. (5) defines the damage incurred by the soil at any time t and for any given applied load τ. The structural damage increases as the applied stress increases. The significance of Eq. (5) rises from the fact that all parameters except A have a quite definite physical meaning. The initial degree of damage ω_o and the constant A can be evaluated through microscopic investigation of a soil sample. Due to the difficulty in determining the microscopic data, the values of ω_o and A are often obtained from conventional creep test. This represents that the constants, ω_o, and Λ were included originally for predicting the change of the structural damage ω. If soil is initially not damaged, then $\omega_o=0$, and Eq. (5) becomes

$$\omega = 1 - \frac{1}{(1 + t)^{\frac{A\tau}{\tau_o - \tau}}} \qquad (6)$$

Eq. (6) indicates the degree of structural damage ω under any applied load τ and at any elapsed time t. One approach for incorporating the damage law into the proposed model is the concept of net stress. The damage accumulates and the amount of material available for carrying the applied load is reduced thus net stress increases. Based on this proposition, the stress are given by Eq. (7).

$$\overset{\vee}{\sigma}_{ij} = \frac{\sigma_{ij}}{1 - \omega} \qquad (7)$$

2.2 Simplification of Generalized Viscous Theory

In the generalized viscous theory, it is assumed that only the plastic response is rate sensitive and the total

strain rate can be resolved into elastic $\dot{\varepsilon}^e_{ij}$ and inelastic viscoplastic $\dot{\varepsilon}^{vp}_{ij}$ parts(Eq. 8), given by Eq. (9)[5].

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^{vp}_{ij} \tag{8}$$

$$\dot{\varepsilon}^{vp}_{ij} = \langle \Phi(F) \rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{9}$$

where the F represents the initial yield function(the static yield function) and the f defines the dynamic loading surface. The viscoplastic strain rate tensor is assumed a function of the "overstress" above the initial yield condition and its magnitude is controlled by the overstress flow function $\langle \Phi(F) \rangle$, two forms of which commonly used are as Eq. (10). The n is a model parameter.

$$\Phi(F) = F^n \quad \text{or} \quad \Phi(F) = \exp F - 1 \tag{10}$$

The stress point can be outside the initial yield surface, instead, the dynamic loading surface passes through the loading point. On this surface, the viscoplastic strain rate is not zero, and its magnitude depends on the overstress flow function $\Phi(F)$. Its direction is given by the gradient vector $\partial f / \partial \sigma_{ij}$, and that like in the associated flow rule of inviscid plasticity, is in the outward normal direction of the dynamic loading surface. The initial yield surface evolves exactly as in plasticity and serves only to separate the region of stress space where deformation is both elastic and viscoplastic from the only elastic region. The dynamic loading surface evolves similarly to the initial yield surface but it is also dependent on the rate of loading in addition to stress and strain.

In this study, the generalized viscous theory stated above is revised as: the initial yield surface and the dynamic loading surface are not differentiated so only one loading surface exists to separate elastic deformation at the stress state inside the surface from both elastic and viscoplastic deformation at the stress state on the surface. The loading surface has the exactly same functional form and hardening rules with the plastic loading function. The right side of Eq. (8), with no model parameter, was used

as the overstress flow function.

4. Model Prediction and Discussion

Figs. 2 and 3 show the model predictions and the experimental results from a series of undrained creep tests for undisturbed normally consolidated clay from Osaka, Japan[8]. The specimens were consolidated to a vertical pressure 3.0kg/cm^2 for 24hours. Then, a prescribed deviatoric stress was applied to each specimen in a single increment. The applied deviatoric stress were 1.20, 1.80, 1.99, 2.19, and 2.30kg/cm^2. The creep strain for two of these tests attenuated after some time had elapsed(Fig. 2), while in the other tests(Fig. 3), creep rupture took place, with time to failure ranging from 400 to 18000minutes, depending on the value of the applied deviatoric stress.
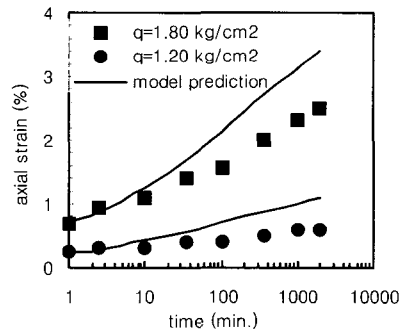

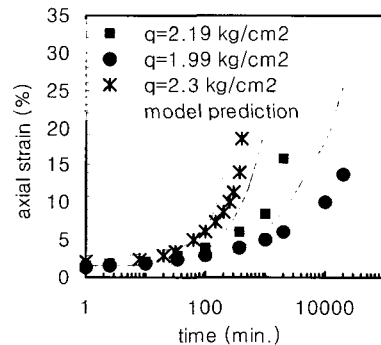**Fig 2.** Creep Behavior(Deviatoric Stress 1.20, 1.80kg/cm^2)

Fig 3. Creep Behavior(Deviatoric Stress 1.99, 2.19, 2.30kg/cm^2)

Values for the input parameters are shown in Table 1[8, 9]. The damage parameter A was determined by

simulating the results of tests that have failed in creep rupture. The instantaneous strength τ_o was taken to be equal to the undrained shear strength for the tested clay.

Table 1. Input parameter

parameter	identification	value
M	slope of CSL	1.47
λ	slope of NCL	0.343
κ	slope of swelling line	0.105
ν	poisson's ratio	0.3
sv	viscous nucleus	1.83
η	viscous parameter	3.5×10^5
n	viscous parameter	8.0
τ_o	damage parameter	3.0
Λ	damage parameter	1.2×10^{-6}

Considering the simplicity of the model, the predictions are generally in good agreement with the experimental results. The respective consideration of the initial yield surface and the dynamic loading surface might be another development regarding the viscous constitutive modeling. The contribution of the modified Cam Clay model could be appreciated in that the isotropically and normally consolidated specimen, on which the model is principally based, was used.

It could be noted that the inclusion of the damage law produced good prediction for the creep rupture cases; however, as the applied deviatoric stress becomes larger than 2.19kg/cm^2, namely over 73% of the undrained shear strength τ_o, it was not possible to obtain a good simulation of creep tests using the same value of the damage parameter in Table 1. It is necessary to use larger value for the damage parameter Λ to acquire better prediction. Instead, it might be more realistic to think that, for those specimens which failed in a relatively short time, they have incurred damage before creep tests began. This is obvious if we notice, for example, that as the applied load increases, just from 1.99 to 2.30kg/cm^2, the time to failure reduced from 18,000 to 400minutes. In other words, 15.5% increase in the applied load led to about a 98% reduction in the time to failure. Thus a more likely possibility is that the damage was actually substantiated during the application of the deviatoric stress, and not after the initiation of the creep test. If this proposition is true, then instead of considering the soil to be undamaged before creep test starts, and the initial damage parameter ω_o should assume a none-zero value.

For the prediction of the creep test under 2.30kg/cm^2 stress level in Fig. 3, the initial damage parameter ω_o was assigned a value of 0.0015, while the values of the other parameters are the same with those in Table 1. As would be expected, the inclusion of the initial damage led to an overestimation of creep strain for the initial part of the creep curve instead of producing relatively better simulation near creep rupture.

The identification and the determination of the model parameter is practically most important in the usage of a constitutive model. In the sense, an elastic-plastic-viscous constitutive equation, in this study, was developed from the point of view that fewer parameters better be employed. It is undesirable that the value of parameter Λ, in this study, was determined from the best match of the test whose result was predicted using the parameter value, even though the parameter value was used to simulate the results of the other tests. That is proper for the case that the specimen from a construction site is tested then the parameter values determined from the test results are used to predict the soil behavior due to the actual construction.

3. Conclusions

In this study, an elastic-plastic-viscous constitutive model was microscopically developed using the generalized Hooke's law, the modified Cam Clay model, the generalized viscous theory simplified, and a damage law. The model was used to simulate the creep behavior of the cohesive soils with or without creep rupture. Comparing the model prediction with the experimental result, the following conclusions can be made.

(1) The proposed constitutive model generally showed good predicting capability for undrained creep strain for normally consolidated cohesive soils.

(2) The incorporation of the damage law made it possible for the model to satisfactorily simulate undrained creep rupture under different stress levels, which shows that the idea of damage law offers a promise in modeling the important creep rupture phenomenon of cohesive soils.

(3) The modified Cam Clay model improved the model accuracy since it is fundamentally based on the isotropically and normally consolidated clay.

(4) The generalized viscous theory was successfully simplified for the simulation of the creep behaviors adopted in this study.

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