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Verification of Single Hardening Model

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단일 경화 모델의 검증

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Abstract In this study, the single hardening model with stress history-dependent plastic potential, which has been most recently proposed based on the critical state soil mechanics and needs few model parameters, was verified for the normally, lightly, and heavily over-consolidated clayey specimens. The triaxial compression tests were strictly conducted. The predictions using the single hardening model generally agree with the measurement. The discrepancy exists on its main focusing on the principal stress rotation; however, the plastic work H and the principal stress rotation angle β are found to be effective indicators of loading history for the plastic potential function of the stress path dependent materials.

Key Words : Single Hardening, Constitutive Model, Plastic Potential, Loading History

요 약 본 연구에서는 응력이력 의존적 소성포텐셜을 도입한 단일 경화 모델의 검증을 정규압밀, 약간 과압밀, 심한 과압밀 점토시료에 대하여 수행하였다. 단일 경화 모델은 한계상태 토질역학을 기본으로 비교적 적은 매개변수를 채택하여 최근에 개발된 탄소성 모델이다. 삼축압축시험이 정밀하게 수행되었으며 시험결과는 모델의 예측결과와 전반적으로 잘 부합하였다. 시험결과와의 차이는 단일 경화 모델이 주로 주응력 회전을 강조하기 때문이다. 그러나 소성일 H 및 주응력 회전각 변수를 통하여 지반재료에서 응력이력을 효과적으로 소성일에 반영할 수 있다는 사실이 검증되었다.

1. Introduction

Various models such as the Cam-Clay models, the Cap models, and the Bounding Surface models, based on the critical state theory, have tried to precisely predict stress-strain behaviors of soils[1]. It is well known that the magnitude and orientation of the principal stresses acting on soil deposits constantly change for a variety of field loading conditions. To better understand the fundamental behavior of soil, the efforts to involve the effect of change in magnitude and orientation of principal stresses, in addition to the effect of loading history, in constitutive modeling need to be made (Lin et al., 2006)[3].

In this study, the single hardening model, which adopts a loading history-dependent plastic potential function and has been recently proposed with relatively few model parameters by Lin et al.(2006), was verified in its prediction capability of stress-strain behavior for normally, slightly, and heavily over-consolidated clayey specimens. This is the verification of the possibility that the model, which has a strong theoretical background of critical state soil mechanics and convenience in use of relatively few model parameters, can simulate over-consolidated clay behavior which has not yet been reliably predicted. The strict triaxial tests were conducted and the experimental results were compared and investigated with the model predictions [4, 5].

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2. Single Hardening Model with Loading-History-Dependent Plastic Potential Function

The single hardening elasto-plastic model including a loading-history-dependent plastic potential function was proposed for kaolin clay subjected to principal stress rotation by Lin et al. (2006)[3]. The model adopts the idea that the effect of change in magnitude and orientation of principal stresses should be investigated. The Young's modulus is expressed stress-dependent(Eq. 1) (Lade and Nelson, 1987)[2], where p_a=atmosphere pressure, I_1=first stress invariant, J_2'=second invariant of deviatoric stress, ν=Poisson's ratio, and M=slope of the critical state line, and λ=slope of normally consolidation line. For each β (defined as the angle between the direction of the major principal stress and the axis of the rotational symmetry, usually vertical direction, Fig. 1), the calculated plastic strain showed yielding of soil and thus increase in the plastic work W_p from the beginning of shearing. The value of β could be expressed as a function of the current stress state using the following equation:

$$E = Mp_a\left[\left(\frac{I_1}{p_a}\right)^2 + R\frac{J_2'}{p_a^2}\right]^\lambda \quad R = 6\frac{1+\nu}{1-2\nu}$$

$$\cos 2\beta = \frac{\sigma_z - \sigma_r}{\sqrt{(\sigma_z-\sigma_r)^2 + 4\tau_{\theta z}^2}} \qquad (1)$$

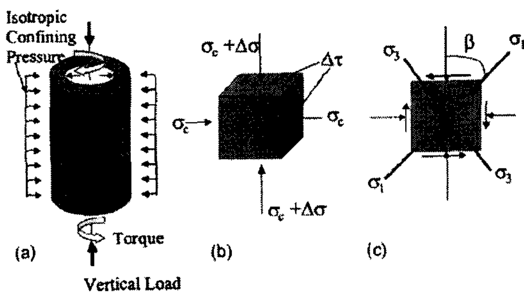

Fig. 1 Loading Scheme and β in axial-torsional test (Lin et al., 2006)[3]

Considering the influence of effective confining pressure, the plastic work evolution during shearing is assumed to be proportional to the total plastic work W_p^o induced during the consolidation stage before shearing. The normalized plastic work evolution during shearing defines the material hardening. The ratio between J_2' and I_1^2 is defined as r, which is similar to the coefficient of friction; therefore, a failure criteria was proposed using a second order polynomial in Eq. (2). In Fig. 2, the hardening variable H is plotted against ratio r/r_f for five β values.

$$H = W_P / W_P^0 - 1 \qquad r = J_2' / I_1^2$$

$$r_f = \frac{J_2'}{I_1^2}\,(at\,failure) = \sum_{i=0}^{2}\alpha_i(\cos 2\beta)^i \qquad (2)$$

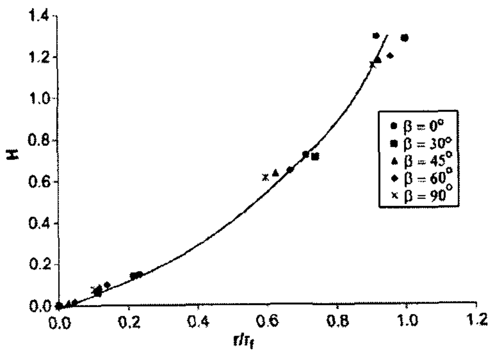

Fig 2. Yield Function and Hardening Rule (Lin et al., 2006)[3]

The relation between H and r/r_f can be approximated by a third order polynomial; therefore, a yield function f and hardening rule H are obtained as below $f = \sum_{i=1}^{3}\xi_i\left(\frac{r}{r_f}\right)^i = H$. Although the coaxiality is typically assumed, the effect of noncoaxiality is evident from the experimental data (Pradel et al., 1990)[4]. The plastic potential could not only be a function of the current stress state but also the loading history (Vardoulakis and Sulem, 1995)[5]. The plastic potential function in a quadratic form could be further simplified as the loading situation applied in the case of the conventional triaxial compression loading scheme (Eq. 3), where a_{ii} are the parameters, which are traditionally independent of stress state, stress paths, and loading history but they are dependent on loading history. In this model, two variables, the hardening variable H and principal stress rotation angle β, were used to reflect the loading history, and the relationships between g, H, and β were constructed based on the experimental observation.[5]

$$g=a_{11}\sigma_z^2+a_{22}\sigma_r^2+a_{33}\sigma_\theta^2+2a_{12}\sigma_z\sigma_r+2a_{13}\sigma_z\sigma_\theta+2a_{23}\sigma_r\sigma_\theta+2a_{44}\sigma\tau_{zr}^2+2a_{55}\tau_{\theta z}^2+2a_{66}\tau_{r\theta}^2 \qquad g=a_{11}\sigma_z^2+a_{22}\sigma_r^2 \tag{3}$$

Since the plastic potential dictates the direction of the plastic strain increment vector, without loss of generality, we can set a_{11}=1. Based on the flow rule, the rest of the constants can be calibrated and rewritten as using Eq. (4). $\epsilon_z^p/\epsilon_r^p$ is simply the slope of the $\epsilon_z^p-\epsilon_r^p$ curve and is related to the variable H by a linear function.

$$\{d\epsilon^p\}=\langle d\lambda\rangle\left\{\frac{\partial g}{\partial\sigma}\right\} \quad \left(\frac{\partial g}{\partial\sigma_z}\right)\Big/\left(\frac{\partial g}{\partial\sigma_r}\right)=\frac{d\epsilon_z^p}{d\epsilon_r^p}$$

$$\left(\frac{\partial g}{\partial\sigma_z}\right)\Big/\left(\frac{\partial g}{\partial\sigma_r}\right)=b_1H+b_2 \tag{4}$$

The slope and the intercept in Eq. (4) vary as β changes. To obtain constants a_{ii} in Eq. (3) for other loading schemes, more equations than $g=H$ are needed. It is apparent that a_{ii} depends on the current stress state and stress path (represented by $cos2\beta$) as well as loading history (reflected by H). In order to obey the second law of thermodynamics and the condition of irreversibility, the plastic potential function has to satisfy the following dissipation inequality $\sigma_{ij}\cdot d\epsilon_{ij}^p\geq 0$. The plastic strain increment $d\epsilon_{ij}^p$ can be obtained by the flow rule and derivatives of the plastic potential function. Since the load factor $\langle d\lambda\rangle\geq 0$ for plastic loading, Eq. (4) can be reduced to Eq. (5).

$$\sigma_{ij}\cdot\frac{\partial g}{\partial\sigma_{ij}}\geq 0 \qquad 2a_{11}\sigma_z^2+2a_{22}\sigma_r^2\geq 0 \qquad 2g\geq 0 \tag{5}$$

The parameters to fully describe the model for the conventional triaxial compression loading condition are the traditional critical state parameters M, v, λ, failure criteria parameters α_0, α_1, α_2 (Eq. 2), yield function parameters ξ_1, ξ_2, ξ_3 (the second row below Fig. 2), and plastic potential function parameters b_1, b_2(Eq. 4). The values of the critical state parameters(M, v, λ) are determined directly from the experimental results as in the standard method in critical state soil mechanics, and the other parameter values are determined from the best-fit of the experimental results.

3. Verification of the Model

The kaolin specimen was prepared for the conventional triaxial compression tests through the slurry consolidometer technique, from which very homogeneous, reproductible, and undisturbed specimens could be prepared with known stress history. The triaxial tests were performed for the normally, slightly (OCR=2), and heavily over-consolidatated (OCR=10) specimens. Table 1 shows the values of the model parameters, whose identification and determination were specified earlier in section 2. Figs 3 to 4 indicate the model predictions and the test results.

Table 1. Model parameters

Parameter	Category	Value
M	Critical State	0.95
v		0.3
λ		0.268
α_0	Failure Criteria	0.0395
α_1		0.0189
α_2		0.0095
ξ_1	Yield Function	0.546
ξ_2		0.475
ξ_3		0.322
b1	Plastic Potential Function	0.604
b2		-2.57

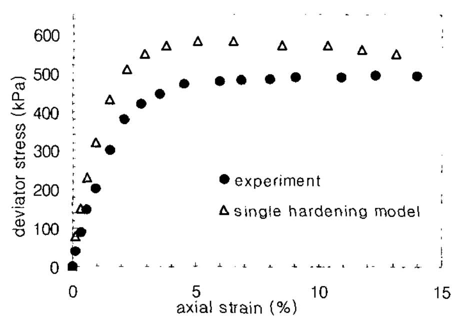

Fig 3. Model Prediction and Test Result(OCR=1)

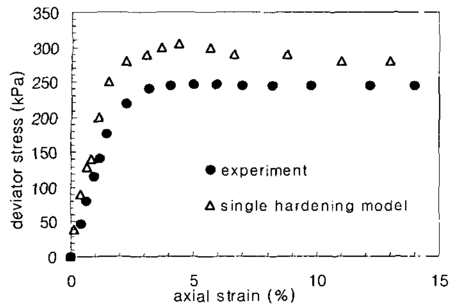

Fig 4. Model Prediction and Test Result(OCR=2)

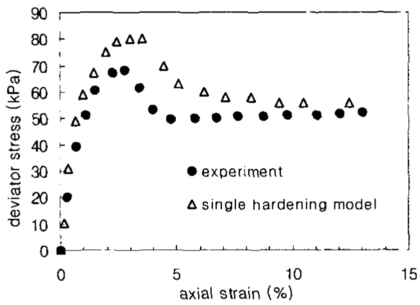

Fig 5. Model Prediction and Test Result(OCR=10)

In Figs. 3 through 5, the predictions using the single hardening model agree with the measurement to some extent, and the discrepancy between the predictions and the experiment is not as large as in the other constitutive models. This is because the single hardening model focuses mainly on the effects of principal stress rotation and the loading history-dependent plastic potential function. The orientation of principal stresses is not be rotated in a conventional triaxial test; however, the model showed its ability to reflect the coaxiality for β=0°. It needs to be noted that the yield function of the single hardening model principally does not simulate the yielding behavior under an isotropic confining stress state because ratio r and thus f become zero in that case. The model considers the monotonic and proportional anisotropic loading conditions and emphasizes the influence of principal stress rotation. Considering the cross-anisotropic nature of clay, the plastic potential function was defined using the invariants of transversely isotropic stress tensor.

In the single hardening model, the volumetric hardening of the clay during the consolidation stage is considered in the form of plastic work evolution before shearing, which is used to normalize the growth of yield surface during plastic deformations and thus the stress-strain response of the .clay. The model considers that the plastic volumetric strains during shearing can also be normalized by the initial plastic work. Hence, the model assumes that the normalized plastic work hardening will implicitly incorporate the volumetric hardening.

Most geomaterials are pressure sensitive and frictional. Therefore, the first stress invariant I_1 and the second deviatoric stress invariant J_2' should be preferably considered as the controlling factors when yield functions and failure criteria are developed. A plastic potential function with a relatively general mathematic function might be employed, such as Eqs. (3) to (5), with varying parameters a_i in each loading step. To identify the way of determining these parameters, one should focus on the relationships between the plastic strain components obtained from experiments because these relationships reflect the variation of the plastic potential function. Once the loading history-dependent plastic potential function is gained, its value can be evaluated at each loading step and the increments of plastic strain components will be obtained using the flow rule Eq. (4). With the flexibility of a loading history-dependent plastic potential function, the behavior of materials under complex loading conditions could be simulated especially for the materials showing highly nonuniqueness of plastic potential. It is apparent that for stress path-dependent materials the plastic potential function in an elasto-plastic model should be a function of loading history. The plastic work H and the principal stress rotation angle β are found to be effective indicators of loading history. Considering the discrepancy between the predictions and the experimental results in the region of large axial strains, more effective method of cooperating the critical state parameter M into the model needs further studies.

4. Conclusions

In this study, the single hardening model, recently developed based on the critical state theory, were investigated by comparing with the experimental results of the conventional compression test for the over-consolidated as well as normally consolidated kaolin clay specimens. The predictions using the single hardening model agree with the measurement to a extent. The discrepancy exists because it is focusing on the effects of principal stress rotation; however, the model showed its ability to reflect the coaxiality for $\beta=0^\circ$ of the conventional triaxial compression and stress history of over-consolidation through the loading history-dependent plastic potential function.

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