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Analysis and Case Study of a K-Stage Inspection System Considering a Re-inspection Policy for Good Items

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양품재검사정책 하에서의 K단계 검사시스템의 분석과 사례연구

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Abstract In this paper, we address a design problem and a case study of a K-stage inspection system, which is composed of K stages, each of which includes an inspection process and a rework process. Assuming the type I and II errors of inspectors and the re-inspection policy for items classified as good, we determine the smallest integer of K which can achieve a given target defective rate. If K does not exist, holding the current values of the type I, II errors, we search reversely a new vector, (the defective rate of an assembly line, the defective rate of a rework process), which can give the target defective rate. Our formulas and methodology based on our K-stage inspection system could be applied and extended to similar situations with slight modifications.

Key words : Multiple inspection system, Defective rate

요 약 본 논문에서는 단계별로 검사공정과 재가공공정으로 구성된 K단계 검사시스템의 설계문제를 다루며 사례연구를 제시한다. 검사원의 Type I과 II의 오차를 가정하고, 양품으로 분류된 제품도 재검사를 한다는 정책 하에 목표 불량률을 달성할 수 있는 가장 작은 정수인 K값을 결정한다. 만약 K값이 존재하지 않을 경우 Type I과 II의 오차는 변하지 않는다고 가정하여 목표 불량률을 달성할 수 있는 (조립생산라인의 불량률, 재가공불량률)을 역으로 탐색한다. 본고에서 제시된 K단계 검사시스템에 대한 공식과 방법론은 약간의 수정을 가한다면 유사한 검사환경에 적용할 수 있다.

1. INTRODUCTION

One of the current hot issues focused by BLU (Back-Light Unit) suppliers is to reduce the average defective rate or the average outgoing rate (AOQ) of BLU's to support the 6σ quality policies currently implemented by LCD (Liquid Crystal Display) suppliers. If an AOQ of BLU's is lower than a demanded defective rate, a BLU supplier may expect economical and managerial savings. On the other hand, if an AOQ becomes higher, a BLU supplier must wait for various disadvantages and may be even deprived of all its rights as supplier. Hence, the strategy for reducing the outgoing quality rate has been recently one of survival strategies [4].

In order to improve AOQ, BLU suppliers must reduce basically the defective rates of their assembly lines. However, the war against FM's (foreign materials such as dusts and threads), which are the major factor in defective rates, has a limit that can not be exceeded. In case of one of BLU suppliers, a single inspector is assigned at the end of a BLU assembly line. If a finished BLU is accepted as good, it is sent to a packing process for a lot operation. Otherwise, it is sent to a rework process. After reworking, it is sent to a packing process without the second inspection. The sampled defective rate just before packing was measured approximately 15,300 PPM and could not attain the goal defective rate, 8,000 PPM, demanded by a

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BLU consumer. Even though the BLU supplier knew the basic and fundamental activities to attain the outgoing defective rate given by a consumer, the simple and quick method was just to select a good BLU well. Thus, the BLU suppliers would like to know how to design and operate their inspection system as well as how to forecast in advance the sampled defective rate at the packing process and the amount of rework.

Since most of papers related with a multiple inspection system assume different designs and operations in addition to limited constraints, and suggest their conclusions, it is not easy to search and utilize the published results from previous papers. For reader's reference, we summarize some papers slightly related with K-stage inspection system as below. Raz and Thomas [1] presented a branch-and-bound method for determining an optimum sequencing inspection plan for a group of inspectors operating at different skill and cost levels. Production and inspection costs for both accepted and rejected items were considered, and dependencies among successive inspections were permitted. Jaraiedi et. al. [2]presented a model which could be used to determine the average outgoing quality for a product which had multiple quality characteristics and which was subject to multiple 100% inspections where the inspection was subject to both type I and type II inspection errors. Assuming a fixed sequence of unreliable inspection operations with known costs and inspection error probabilities of two types, Avinadav and Raz [3]developed a model for selecting the set of inspections that should be activated in order to minimize expected total costs (inspection and penalties), and provided an efficient branch-and-bound algorithm for finding the optimal solution.

In this paper, by extending an existing BLU inspection system mentioned above, we address a design problem and a case study of the K-stage inspection system assuming the re-inspection policy for items classified as good. We determine the smallest integer of K which can achieve a given target defective rate demanded by consumers. If K does not exist, holding the type I, II errors, we search reversely a new vector, (the defective rate of an assembly line, the defective rate of a rework process), which can give the target defective rate.

In Section 2, we describe our problems for a K-stage inspection system in detail. In Section 3, we derive a formula for the average defective rate just before packing as a function of five factors (type I error, type II error, the defective rate of an assembly line, the defective rate of a rework process, and K). In addition, we provide a formula for finding a minimum integer of K so that the target defective rate demanded by a consumer is attained. Since the nonexistence of a minimum value of K indicates that the target defective rate can not be attained, we should search new combination of factors in order to attain it. Hence, assuming that the vector, (type I error, type II error), is fixed, we search and find anew vector (the defective rate of an assembly line, the defective rate of a rework process, and K) which can give the target rate. In Section 4, a case study will be given and analyzed.

2. PROBLEMS STATEMENT

As shown in Figure 1, our K-stage inspection system consists of K stages, each of which includes an inspection process and a rework process. In the first stage, if an item coming off from an assembly line is classified as good by the first inspector, then it is sent to the second inspection process. Otherwise, it is immediately reworked by the first reworker (Practically speaking, both inspection and rework can be made by the same operator) and, then it is sent to the second inspection process. In the second stage, items are treated in the same manner and so on. In the last K-th stage, an item classified as good is sent to the packing process and an item classified as bad is reworked and immediately sent to the packing process without inspection.

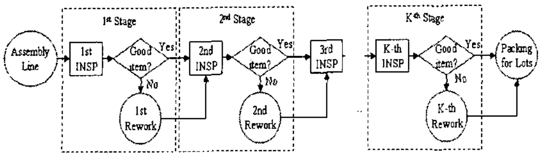


Fig 1. A process diagram of our K-stage inspection system

We assume that inspectors make two kinds of errors of judgment; rejecting a gooditem (type I error) or accepting a bad item (type II error). Let α and β be the probabilities of a type I and type II errors by an inspector respectively. Let q_0 and q_R be the average defective rate of items coming off from an assembly line and the average defective rate at a rework process respectively, and without loss of generality assume that $q_0 > q_G$ where q_G is a given target defective rate. Then, the average defective rate of items at the packing process, denoted by q_K, can be expressed as a function of a vector (α,β,q_0,q_R,K), i.e., $f(\alpha,\beta,q_0,q_R,K)$. Our objective is to find the smallest integer of K, denoted by K^*, such that $\hat{q}_K = f(\hat{\alpha},\hat{\beta},\hat{q}_0,\hat{q}_R,K) \le q_G$ where $(\hat{\alpha},\hat{\beta},\hat{q}_0,\hat{q}_R)$ is the estimate of (α,β,q_0,q_R). In other words, our first problem can be stated as follows:

INSP : Given $(\hat{\alpha},\hat{\beta},\hat{q}_0,\hat{q}_R,q_G)$, find K such that we
Minimize K
subject to $\hat{q}_K = f(\hat{\alpha},\hat{\beta},\hat{q}_0,\hat{q}_R,K) \le q_G$

In case that K^* does not exist or when an alternative for (q_0,q_R,K) is needed, it is natural to search new combination of (α,β,q_0,q_R,K) in order to achieve q_G. Since finding those values of a new vector is very complicated, assuming that $(\hat{\alpha},\hat{\beta})$ does not change, we search $(\tilde{q}_0,\tilde{q}_R,\tilde{K})$ so that the average defective rate of items at the packing process, denoted by $\tilde{q}_{\tilde{K}}$, may achieve q_G. That is,

SP : Given $(\hat{\alpha},\hat{\beta},q_G)$, find $(\tilde{q}_0,\tilde{q}_R,\tilde{K})$ such that $\tilde{q}_{\tilde{K}}$
$= f(\hat{\alpha},\hat{\beta},\tilde{q}_0,\tilde{q}_R,\tilde{K}) = q_G$

3. ANALYSIS OF OUR INSPECTION PROBLEMS

3.1 Derivation of q_K and Some Properties

As shown in Figure 2, for a positive integer k, let G_k and B_k be the numbers of good and bad items given to the (k+1)-th inspection stage respectively. Let R_0 be the number of items coming into the first stage of our K-stage inspection system and let G_0 and B_0 the numbers of good and bad items given to the first inspection stage respectively. Clearly, $G_0 = (1-q_0)R_0$ and $B_0 = q_0 R_0$. The numbers used in this paper are assumed to be real since R_0 is big enough to justify this assumption.

Let $N_{G/G}(k)$ and $N_{B/G}(k)$ be the numbers of good items that are judged as good and bad by the k-th inspector respectively. Let $N_{G/B}(k)$ and $N_{B/B}(k)$ be the number of bad items that are judged as good and bad by the k-th inspector respectively. Then, for a positive integers k, $N_{G/G}(k) = (1-\alpha)G_{k-1}$, $N_{B/G}(k) = \alpha G_{k-1}$, and $N_{G/B}(k) = \beta B_{k-1}$ and $N_{B/B}(k) = (1-\beta)B_{k-1}$.

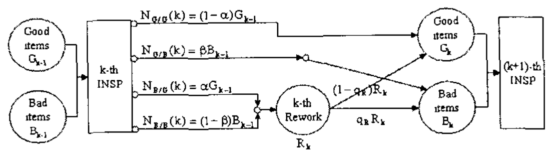


Fig. 2. A flow diagram for the number of good and bad items classified after the k-th inspection

Let R_k be the number of items sent to the k-th rework process. Since R_k is the sum of the number of good items classified as bad and the number of bad items classified as bad, R_k can be expressed as

$$R_k = N_{B/G}(k) + N_{B/B}(k) = \alpha G_{k-1} + (1-\beta)B_{k-1}$$
for k=1, 2, ⋯, K (1)

After reworking at the k-th rework process, since G_k becomes the sum of the number of good items classified as good and the number of good reworked items, we have

$$G_k = N_{G/G}(k) + (1-q_R)R_k = (1-\alpha)G_{k-1} + (1-q_R)R_k$$
for k=1, 2, ⋯, K (2)

Similarly, we have

$$B_k = N_{G/B}(k) + q_R R_k = \beta B_{k-1} + q_R R_k$$
for k=1, 2, ⋯, K (3)

Since we assume that all the items passing through the K-th rework process are sent to the packing process without inspection, the average defective rate at the packing process can be expressed as

$$q_K = \frac{B_K}{G_K + B_K} = \frac{\beta B_{K-1} + q_R R_K}{G_K + B_K}$$
for K=0, 1, 2, ⋯ (4)

In order to derive an analytical expression for q_K, we need to prove the following properties. For convenience, let τ and ρ be $(1-\alpha)q_R + \beta(1-q_R)$ and $\alpha q_R(1-\tau)^{-1}$ respectively. Without loss of generality, we assume that $0<\alpha,\beta,q_0,q_R<1$.

Property 1. For a nonnegative integer k, $G_k + B_k = R_0$

Proof : By definition, $G_0 + B_0 = R_0$. For a positive integer k, we have

$$G_k + B_k = (1-\alpha)G_{k-1} + (1-q_R)R_k + \beta B_{k-1} + q_R R_k$$
(Using Eq.(2) and Eq.(3))
$$= (1-\alpha)G_{k-1} + \beta B_{k-1} + \alpha G_{k-1} + (1-\beta)B_{k-1}$$
(Using Eq.(1))
$$= G_{k-1} + B_{k-1}$$

It follows that $G_k + B_k = G_0 + B_0 = R_0$. □

Property 2. If $0<\alpha,\beta,q_R<1$, then $0<\tau<1$.

Proof : Since $0<\alpha,\beta,q_R<1$, it follows that $0<\tau$.

If $\beta \ge 1-\alpha$, $\tau = (1-\alpha)q_R + \beta(1-q_R)$
$$\le \beta q_R + \beta(1-q_R) = \beta < 1.$$
If $\beta < 1-\alpha$, $\tau = (1-\alpha)q_R + \beta(1-q_R)$
$$\le (1-\alpha)q_R + (1-\alpha)(1-q_R) = 1-\alpha < 1.$$

Therefore we have, $0<\tau<1$. □

Property 3. If $0<\alpha,\beta,q_R<1$, then

(1) $G_k = R_0\{(1-\rho)-(q_0-\rho)\tau^k\}$,
for k=0, 1, 2, ⋯, K

(2) $B_k = R_0\{\rho + (q_0-\rho)\tau^k\}$,
for k=0, 1, 2, ⋯, K

(3) $R_k = R_0\{(1-\alpha-\beta)(q_0-\rho)\tau^{k-1} + \alpha(1-\rho) + (1-\beta)\rho\}$,
for k=1, 2, ⋯, K

Proof : (1) For k=1, 2, ⋯, K, we have,

$$G_k = (1-\alpha)G_{k-1} + (1-q_R)R_k$$
(Using Eq.(2))
$$= (1-\alpha)G_{k-1} + (1-q_R)\{\alpha G_{k-1} + (1-\beta)B_{k-1}\}$$
(Using Eq.(1))
$$= (1-\alpha q_R)G_{k-1} + (1-q_R)(1-\beta)B_{k-1}$$
$$= (1-\alpha q_R)G_{k-1} + (1-q_R)(1-\beta)(R_0 - G_{k-1})$$
(Using Property 1)
$$= \{(1-\alpha)q_R + \beta(1-q_R)\}G_{k-1} + (1-q_R)(1-\beta)R_0$$
$$= \tau G_{k-1} + (1-q_R)(1-\beta)R_0$$

Since $\tau \ne 1$ from Property 2, solving the above first order linear difference equation and letting ρ be $\alpha q_R(1-\tau)^{-1}$, we have

$$G_k = \left\{G_0 - \frac{(1-q_R)(1-\beta)R_0}{1-\tau}\right\}\tau^k + \frac{(1-q_R)(1-\beta)R_0}{1-\tau}$$
$$= R_0\left(-q_0 + \frac{\alpha q_R}{1-\tau}\right)\tau^k + R_0\left(1 - \frac{\alpha q_R}{1-\tau}\right)$$

$(\because (1-q_R)(1-\beta)=1-\tau-\alpha q_R)$

$=R_0\{(1-\rho)-(q_0-\rho)\tau^k\}$

Since the above equation holds true when k=0, (1) holds true for a nonnegative integer k.

(2) Since $B_k=R_0-G_k$,

we have, $B_k=R_0\{\rho+(q_0-\rho)\tau^k\}$.

Since the above equation holds true when k=0, (2) holds true for a nonnegative integer k.

(3) For k=1, 2, ⋯, K, we have,

$R_k=\alpha G_{k-1}+(1-\beta)B_{k-1}$ (From Eq.(1))

$=R_0\{(1-\alpha-\beta)(q_0-\rho)\tau^{k-1}+\alpha(1-\rho)+(1-\beta)\rho\}$

(Using Property 3-(1) and (2)) □

Proposition 4. If $0<\alpha,\beta,q_R<1$, then for a nonnegative integer K,

(1) $q_K=\rho+(q_0-\rho)\tau^K$

(2) $\lim_{K\to\infty} q_K=\rho$

Proof : (1) Using Eq.(4), Property 1, and Property 3-(2), for a nonnegative integer K, we have

$$q_K=\frac{B_K}{G_K+B_K}=\frac{B_K}{R_0}=\rho+(q_0-\rho)\tau^K$$

(2) Since $0<\tau<1$, $\lim_{K\to\infty}\tau^K=0$. It follows that $\lim_{K\to\infty} q_K=\rho$. □

If inspectors are perfect, that is, $(\alpha,\beta)=(0,0)$, then using Proposition 4, $q_K=q_0q_R^K$ and q_K converges to zero. On the other hand, when inspectors are imperfect, that is $(\alpha,\beta)=(1,1)$, $q_K=1-(1-q_0)(1-q_R)^K$ and q_K converges to one. It follows that the feasible region of q_K can be theoretically represented as the light dark area as shown in Figure 3. It can be observed that our inspection system does not always guarantee that q_K will decreases as K increases. Hence, it will be useful to derive some conditions that guarantee $q_K \le q_G$.

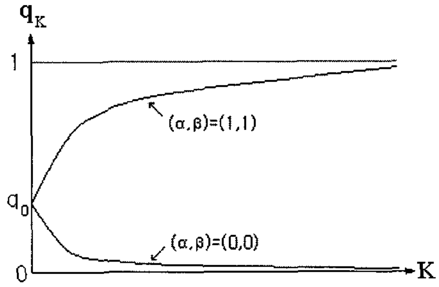


Fig 3. The feasible region of q_K

Proposition 5. If $0<\alpha,\beta,q_R<1$, for a nonnegative integer K,

(1) If $0<q_0<\rho$, then q_K is a strictly increasing function of K. That is, $q_{K+1}<q_K$.

(2) If $q_0=\rho$, then q_K is a constant. That is, $q_{K+1}=q_K=\rho$.

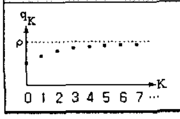
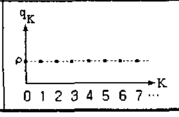
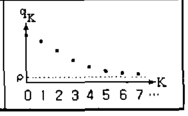
(3) If $\rho<q_0<1$, then q_K is a strictly decreasing function of K. That is, $q_{K+1}<q_K$.

Proof : Using Proposition 4-(1)

$q_{K+1}-q_K=(\rho-q_0)(1-\tau)\tau^K$. Since $(1-\tau)>0$ and $\tau^K>0$, (1), (2), and (3) hold true. □

From Proposition 4 and Proposition 5, the shape of q_K, depending on ρ, will become one of three shapes as shown in Table 1. The case that $0<q_0\le\rho$ is meaningless since q_K will not decrease as K increases. The case that $\rho<q_0<1$ always guarantees that q_K will decreases as K increases.

Table 1. Three shapes of q_k

$0<q_0<\rho$	$q_0=\rho$	$\rho<q_0<1$
		

3.2 Determination of K^*

It can be observed that the case that $\rho < q_0 < 1$ does not always guarantee that $q_K \le q_G$ since there is a case that $\lim_{K\to\infty} q_K \ge q_G$. Thus, if $0 < \alpha, \beta, q_R < 1$, $\rho < q_0 < 1$, and $\lim_{K\to\infty} q_K < q_G$, then from Proposition 4 and Proposition 5, there exists a positive integer K^* such that $q_{K^*} \le q_G$. Now, K^* can be derived as follows. Define $\lceil x \rceil$ to be the smallest integer greater than or equal to x. Let $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R)$ be an estimated vector of $(\alpha, \beta, q_0, q_R)$.

Theorem 6. If $0 < \hat{\alpha}, \hat{\beta}, \hat{q}_R < 1$ and
$\lim_{K\to\infty} q_K = \hat{\rho} < q_G < \hat{q}_0 < 1$, then

$$K^* = \left\lceil \frac{\ln \hat{\omega}}{\ln \hat{\tau}} \right\rceil$$

where $\hat{\tau} = (1-\hat{\alpha})\hat{q}_R + \hat{\beta}(1-\hat{q}_R)$, $\hat{\rho} = \hat{\alpha}\hat{q}_R(1-\hat{\tau})^{-1}$, and $\hat{\omega} = \dfrac{q_G - \hat{\rho}}{\hat{q}_0 - \hat{\rho}}$.

Proof : Given $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, q_G)$, we need to obtain the smallest value of K such that $\hat{q}_K = f(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, K) \le q_G$. Let K_E be the real number such that $f(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, K_E) = q_G$. That is, using Proposition 4-(1), we have $f(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, K_E) = \hat{\rho} + (\hat{q}_0 - \hat{\rho})\hat{\tau}^{K_E} = q_G$.

Since $\hat{\rho} \ne \hat{q}_0$, solving the above equation for K_E gives $K_E = \ln\hat{\omega}/\ln\hat{\tau}$ where $\hat{\tau} = (1-\hat{\alpha})\hat{q}_R + \hat{\beta}(1-\hat{q}_R)$, $\hat{\rho} = \hat{\alpha}\hat{q}_R(1-\hat{\tau})^{-1}$, and $\hat{\omega} = (q_G - \hat{\rho})(\hat{q}_0 - \hat{\rho})^{-1}$. Since $\lim_{K\to\infty} q_K < q_G$ and q_K is a strictly decreasing function of K from Proposition 5-(3), K^* will be the smallest integer greater than or equal to K_E. Hence, we can express K^* as $K^* = \lceil K_E \rceil = \lceil \ln\hat{\omega}/\ln\hat{\tau} \rceil$. □

3.3 Estimation of (α, β)

There might be various methods for estimating (α, β). In this paper, we suggest an indirect method for estimating (α, β) as follows. Two equations can be obtained from Property 3-(3) and Proposition 4-(1) as follows.

$$(1-q_0)\alpha - q_0\beta = \frac{R_1}{R_0} - q_0 \tag{5}$$

$$(1-q_0)q_R\alpha + q_0(1-q_R)\beta = q_1 - q_0 q_R \tag{6}$$

Since it is practically possible to estimate both (q_0, q_1, q_R) and (R_0, R_1), the right hand sides of Eq.(5) and Eq.(6)will be constants. Thus, we can obtain the following estimators by solving simultaneously the above two equations.

$$\hat{\alpha} = \frac{1}{(1-\hat{q}_0)}\left\{\hat{q}_1 - \hat{q}_0 + \frac{\hat{R}_1}{\hat{R}_0}(1-\hat{q}_R)\right\} \tag{7}$$

$$\hat{\beta} = \frac{1}{\hat{q}_0}\left\{\hat{q}_1 - \frac{\hat{R}_1}{\hat{R}_0}\hat{q}_R\right\} \tag{8}$$

3.4 A Procedure for Our Search Problem

Nonexistence of K^* indicates that our K-stage inspection system with the current vector, $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R)$, can not achieve q_G. There might be several ways in order to meet q_G. In this paper, we assume that (α, β) can not be controlled and that we can control (q_0, q_R) from $(\hat{q}_0, \hat{q}_R)$ to $(\tilde{q}_0, \tilde{q}_R)$ such that $f(\hat{\alpha}, \hat{\beta}, \tilde{q}_0, \tilde{q}_R, \tilde{K}) = \tilde{q}_{\tilde{K}} = q_G$ for a positive integer $\tilde{K}$.

In our search problem, $(\tilde{q}_0, \tilde{q}_R)$ can be easily determined by holding $\tilde{q}_R$ since we can derive a linear equation for $\tilde{q}_0$ by holding $\tilde{q}_R$. On the other hand, holding $\tilde{q}_0$ gives a nonlinear equation for $\tilde{q}_R$. By holding $\tilde{q}_R$ and using Proposition 4-(1), we have,

$$\tilde{q}_{\tilde{K}} = \tilde{\rho} + (\tilde{q}_0 - \tilde{\rho})\tilde{\tau}^{\tilde{K}} = q_G \tag{9}$$

where $\tilde{\tau} = (1-\hat{\alpha})\tilde{q}_R + \hat{\beta}(1-\tilde{q}_R)$ and $\tilde{\rho} = \hat{\alpha}\tilde{q}_R(1-\tilde{\tau})^{-1}$.

Hence, we can express $\tilde{q}_0$ as a function of $(\tilde{q}_R, \tilde{K})$ as follows and we can obtain $(\tilde{q}_0, \tilde{q}_R, \tilde{K})$ when $(\tilde{q}_R, \tilde{K})$ is given.

$$\tilde{q}_0 = \tilde{\rho} + (q_G - \tilde{\rho})\tilde{\tau}^{-\tilde{K}} \qquad (10)$$

4. A CASE STUDY

After collecting related data from a BLU supplier for six months, we estimated $(q_0, q_1, q_R, R_0, R_1)$ as (16.1%, 1.53%, 5.0%, 1,200,000 units, 193,000 units). Using Eq.(7) and Eq.(8), we estimated (α, β) as (0.8453%, 4.5083%). Since the target defective rate of the BLU supplier in study was 8,000 PPM, K^* exists in our case study since $\lim_{K\to\infty} q_K$ was computed using Proposition 4-(2) as 466 PPM < 8,000 PPM $= q_G$. The values of $\hat{q}_K$ for K=2, 3, 4, 5 based on our model were computed sequentially as 1,836 PPM, 592 PPM, 477 PPM, 467 PPM as shown in Figure 4, clearly we have $K^*=2$. The same result can be obtained from Theorem 6 since $\hat{\tau}$=9.2406%, $\hat{\rho}$=0.0466%, and $\hat{\omega}$=4.6933%. It can be observed that the value of $\hat{q}_K$ drops with the biggest slope when K=1, and that the falling slope between the successive values of K decreases very slowly as Kincreases.

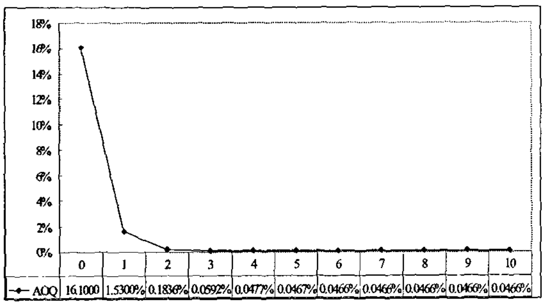


Fig. 4. The value of $\hat{q}_K$ given that $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R)$= (0.8453%, 4.5083%, 16.1%, 5.0%)

For reader's reference, the values of K^* depending on (q_0, q_R) are computed and summarized in Table 3.

Table 2. The value of K^* depending on (q_0, q_R) given that $(\hat{\alpha}, \hat{\beta})$= (0.8453%, 4.5083%)

q_0 \ q_R	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%
5%	1	1	1	2	2	2	3	3	4	6
10%	1	2	2	2	2	3	3	4	5	7
15%	1	2	2	2	3	3	4	4	5	7
20%	2	2	2	3	3	3	4	5	6	8
25%	2	2	2	3	3	4	4	5	6	8
30%	2	2	2	3	3	4	4	5	6	8
35%	2	2	2	3	3	4	4	5	6	9
40%	2	2	3	3	3	4	5	5	7	9
45%	2	2	3	3	3	4	5	6	7	9
50%	2	2	3	3	4	4	5	6	7	9

When K^*=2, the value of $\hat{q}_K$ is computed as 1,836 PPM, which is much lower than 8,000 PPM. As shown in Table 3, as long as $\hat{q}_R$=5%, 8,000 PPM can be achieved by either a 2-stage inspection system with 8.200% < q_0 ≤=88.2817% or a 1-stage inspection system with q_0 ≤=8.2000%. In other words, if a quality control manager would like to reduce the value of K from two to one without changing q_R, he/she might set the target defective rate of q_0 as 8.200%. He/she might set (q_0, q_R) from $(\hat{q}_0, \hat{q}_R)$=(16.1%, 5.0%) to $(\tilde{q}_0, \tilde{q}_R)$=(9.2377%, 4%) or $(\tilde{q}_0, \tilde{q}_R)$=(10.5426%, 3%) and so on. An appropriatechoice among many alternatives may be selected depending upon circumstances of a company.

Suppose that q_G=400 PPM. Since $\lim_{K\to\infty} q_K$=466 PPM, we can not achieve 400 PPM at all using the current values (0.8453%, 4.5083%, 16.1%, 5.0%) of $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R)$regardless of any value of K. As discussed, holding $(\hat{\alpha}, \hat{\beta})$, we search the curve of $(\tilde{q}_0, \tilde{q}_R)$, all the points on which can achieve 400 PPM. By setting some practical combination of $(\tilde{q}_R, \tilde{K})$ for $\tilde{K}$=1, 2, 3 and $\tilde{q}_R$=0%, 1%, ..., 4%, using Eq.(15), we compute and summarize $\tilde{q}_0$'s in Table 4. For example, if the company improves (q_0, q_R) from $(\hat{q}_0, \hat{q}_R)$=(16.1%, 5.0%) to either $(\tilde{q}_0, \tilde{q}_R)$=(2.3667%, 3%) or $(\tilde{q}_0, \tilde{q}_R)$=(5.3719%, 2%), then the given target defective rate of 400 PPM can be theoretically achieved by a 2-Stage inspection system(i.e., $\tilde{K}$=2). Suppose that a 1-stage inspection system must be used and that the current value of $\hat{q}_R$(=5%) can not be reduced. Then, if you can not reduce q_0, the target defective rate of 400 PPM can not be achieved.

Table 3. $(\tilde{q}_0, \tilde{q}_R, \tilde{K})$ when $(\hat{\alpha}, \hat{\beta}, q_G)$=(0.8453%, 4.5083%, 8,000 PPM).
(Entries =$\tilde{q}_0$%, n.e. = nonexistence)

$\tilde{q}_R$	$\tilde{\tau}$	$\tilde{\rho}$	$\tilde{K}$=1	$\tilde{K}$=2
1.0%	5.4548%	0.0089%	14.5111%	n.e.
2.0%	6.4012%	0.0181%	12.2335%	n.e.
3.0%	7.3477%	0.0274%	10.5426%	n.e.
4.0%	8.2942%	0.0369%	9.2377%	n.e.
5.0%	**9.2406%**	**0.0466%**	**8.2000%**	**88.2817%**
6.0%	10.1871%	0.0565%	7.3552%	71.7035%
7.0%	11.1335%	0.0666%	6.6540%	59.2341%
8.0%	12.0800%	0.0769%	6.0627%	49.6281%
9.0%	13.0265%	0.0875%	5.5573%	42.0777%
10.0%	13.9729%	0.0983%	5.1204%	36.0401%

Table 4. $(\tilde{q}_0, \tilde{q}_R, \tilde{K})$ when $(\hat{\alpha}, \hat{\beta}, q_G)$=(0.8453%, 4.5083%, 400 PPM).(Entries =$\tilde{q}_0$%)

$\tilde{q}_R$	$\tilde{\tau}$	$\tilde{\rho}$	$\tilde{K}$=1	$\tilde{K}$=2	$\tilde{K}$=3
0.0%	4.5083%	0.0000%	0.8873%	19.6804%	n.e.
1.0%	5.4548%	0.0089%	0.5783%	10.4475%	n.e.
2.0%	6.4012%	0.0181%	0.3608%	5.3719%	83.7%
3.0%	7.3477%	0.0274%	0.1993%	2.3667%	31.9%
4.0%	8.2942%	0.0369%	0.0746%	0.4918%	5.5%
5.0%	9.2406%	0.0466%	n.e.	n.e.	n.e.

5. CONCLUDING REMARKS

In this paper, assuming the type I, II errors and the re-inspection policy for items classified as good, we provide a formula for determining the smallest value of K which can achieve a given target defective rate. If the value of K does not exist, holding the type I, II errors, we provide a procedure for searching a new vector, (the defective rate of an assembly line, the defective rate of a rework process), which can give the given target defective rate. Our formulas and methodology based on our K-stage inspection system could be applied and extended to similar situations with slight modifications.

Further research may be concentrated on the study for the selecting ability of an inspector due to the psychological warfare or variation, and the study for determining an optimal defective rate in terms of costs and benefits due to the reduction of defective rate.

REFERENCES

[1] Raz, T. and M. U. Thomas, "A Method for Sequencing Inspection Activities Subject to Errors". IIE Transactions, Vol. 15, No. 1, 12-18, 1982

[2] Jaraiedi, M., Kochhar, D. V., and S. C. Jaisingh, "Multiple Inspections to Meet Desired Outgoing Quality", Journal of Quality Technology, Vol. 19, No. 1, 46-51, 1987

[3] Avinadav, T. and T. Raz, "Economic optimization in a fixed sequence of unreliable inspections", Journal of Operational Research Society, Vol. 54, 605-613, 2003

[4] Kim, T. K., A Case Study of Factory Innovation for BLU Manufacturing, Dankook University, Master Thesis, 2004

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