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The Expectation for Material Properties of Microstructure by Application of Dynamic Response Characteristics

Jeong-Ick Lee[1*] and Moon-Su Yeo[2]

동적 응답 특성을 활용한 미세구조의 물성 분포에 대한 예측

이정익[1*], 여문수[2]

Abstract This paper addresses the prediction of the material property continuities of a microstructure. Prediction was made by measuring the dynamic responses distribution of the fabricated materials used in the microstructures. When these distributional material properties were used in estimating the mechanical performances of microstructures, the differences between the computer simulation and the experimental result of microstructures could be reduced and their reliability design could be made.

Key Words : Microstructure(미세구조), Young's Modulus(영의 계수), Shear Modulus(전단 계수), Poisson's Ratio(포아송 비), Natural Frequency(고유 진동수), Reliability Design(신뢰성 설계)

요 약 본 연구는 재료 특성에 있어 미세구조의 연속성을 예측한 것이다. 예측은 미세구조에서 사용되는 제작 재료의 동적 응답 분포를 측정해서 만들어졌다. 분포되는 재료 특성들이 미세구조의 기계적 성능을 평가하는데 사용될 때, 미세구조에 대한 컴퓨터 시뮬레이션과 실험결과의 차이를 줄일 수 있고 신뢰성 설계가 이루어질 수 있다.

1. Introduction

The material properties of a microstructure can not be the same as those of a macrostructure. Material discontinuities, such as porosities and unevenness, exist to some extent in almost all materials. It is very difficult to uniformly predict material properties and behaviors since these material discontinuities are randomly and unevenly distributed. These properties affect the predictions of structural performances in the microstructure additionally, the static and dynamic characteristics can change. For the material compensation in computer-aided simulation, dynamic characteristics such as natural frequency of bending mode and torsion mode may be used in the prediction of material properties. To investigate the effect due to material discontinuities, there have been many studies measuring material discontinuities [1-3]. These have the drawbacks in preparation and installation of the specimens.

The porosity-measuring approach has been proposed for the material discontinuities however, this approach has difficulties in installations and measurement [1].

This paper presents the predictions of the material property distribution due to material discontinuities through the use of dynamic coefficients of natural frequency and numerical relationships. These predictions of distributional material properties could help measure the mechanical structural performances in order to assess the reliability of structure designs. Compensated Young's modulus with proposed method is verified with simulation and experiment of measuring the natural frequency of MEMS gyro sensor.

[1]Dept. of Mechanical Design., Inha Techinical College (Professor)
[2]Dept. of Automotive Eng., Inha Techinical College (Professor)
*Corresponding author : Jeong-Ick Lee(jilee@inhatc.ac.kr)

2. Theoretical Calculation

Generally, the material properties with porosity in the mechanical structure can be given by,

$$E = x^n E_0 \tag{1}$$

where E : the elastic modulus with porosity,

E_0 : the elastic modulus without porosity,

x : the volume density

n : the density index[4-5].

The structural rigidity mechanically depends on the relationship of stress-strain $(\{\sigma\} = [D]\{\varepsilon\})$ and the elastic constants $[D]$ of 2-D and 3-D element for an isotropic material are given by [6],

For the plane stress,

$$D(x) = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{2}$$

For the plane strain,

$$D(x) = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2 \end{bmatrix} \tag{3}$$

For the solid element,

$$D(x) = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{4}$$

The variation of structural rigidity with respect to material porosity can be calculated using the following relationships,

For the static problem,

$$[K]\{u\} = \{F\}$$

$$[K]\frac{\partial\{u\}}{\partial x} = -\frac{\partial[K]}{\partial x}\{u\} = \{\tilde{f}\} \tag{5}$$

where $\{\tilde{f}\}$: the pseudo-load.

For the eigen-value problem,

$$[K]\{\phi\} - \lambda[M]\{\phi\} = 0$$

$$\frac{\partial\lambda_i}{\partial x} = \frac{\{\phi_i\}^T\left(\frac{\partial[K]}{\partial x} - \lambda_i\frac{\partial[M]}{\partial x}\right)\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}} \tag{6}$$

$$= \{\phi_i\}^T\frac{\partial[K]}{\partial x}\{\phi_i\} - \lambda_i\{\phi_i\}^T\frac{\partial[M]}{\partial x}\{\phi_i\}$$

$$= \frac{E'(x)}{E(x)}\{\phi_i\}^T[K]\{\phi_i\} - \frac{1}{x}\lambda_i\{\phi_i\}^T[M]\{\phi_i\}$$

where $\{\phi_i\}^T[M]\{\phi_i\} = 1$. The entries of stiffness matrix $[K]$ can be written by eq. (7).

$$[K] = \int_V [B]^T[D(V)][B]dV \tag{7}$$

where $[B]$ spatial derivative matrix of displacement variables

Porosity can be defined as the fraction of void volume in the material [7-9]. The effects of porosity on Young's modulus have dealt with fitting an empirical curve to actual data for a material. The equations of Young's modulus with porosity from eq. (1) can be given as follows; the linear equation (8), the exponential empirical equation (9) and semi-empirical equation (10).

$$E = (1 - \alpha\bar{V})E_0 \tag{8}$$

$$E = E_0 e^{-b\bar{V}} \tag{9}$$

$$E = E_0\left[1 - \frac{a\bar{V}}{1+(a-1)\bar{V}}\right] \tag{10}$$

where,

α, a and b : the empirical constants

$\overline{V}$: the volume porosity from pore size and distributions $\left(\overline{V} = {}^{V_p}\!/_{V_o}\right)$

V_p : pore volume

V_o : structure volume

α, a, b and $\overline{V}$ can be found from the probability distribution of pore size, cell size and volume size. But the above equations can be given from the well-known physical parameters of empirical constants and volume porosity. This paper presents the estimation method of material properties of microstructure through the vibration analysis of a microstructure. The Young's modulus and shear modulus can stochastically be predicted by the dynamic response of beams [10-12]. Basically, the transverse vibration of the Bernoulli-Euler beams can be assumed as:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \tag{11}$$

where $$y(x,t) = Y(x)\cos(\omega t + \psi) \tag{12}$$

Using a separation of variables approach, the characteristic equation is given by

$$\frac{d^4 Y}{d x^4} - \lambda^4 Y = 0 \tag{13}$$

where, $$\lambda^4 = \frac{\rho A \omega^2}{EI} \tag{14}$$

The general solution of eq. (13) for the deformation mode may be written in the following form:

$$Y(x) = c_1 \sinh\lambda x + c_2\cosh\lambda x + c_3 \sin\lambda x + c_4\cos\lambda x \tag{15}$$

There were five constants in the general solution, that is, four amplitude constants and the eigen-value λ. The end conditions were used for these constants. In the cantilever beam model, the boundary conditions of the deformation mode were given by:

$$Y(0) = 0,\ \left.\frac{dY}{dx}\right|_{x=0} = 0,\ \left.\frac{d^2Y}{dx^2}\right|_{x=L} = 0,\ \left.\frac{d^3Y}{dx^3}\right|_{x=L} \approx 0 \tag{16}$$

From the above boundary conditions, the following equations were obtained:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ \lambda & 0 & \lambda & 0 \\ \lambda^2\sinh\lambda L & \lambda^2\cosh\lambda L & -\lambda^2\sin\lambda L & -\lambda^2\cos\lambda L \\ \lambda^3\cosh\lambda L & \lambda^3\sinh\lambda L & -\lambda^3\cos\lambda L & \lambda^3\sin\lambda L \end{bmatrix}\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{17}$$

The determinant of the coefficients must vanish in order for there to be a nontrivial solution. This led to the characteristic equation,

$$\cos\lambda L\cosh\lambda L = -1 \tag{18}$$

where $(\lambda_n L)$ is the n[th] specific value satisfying the geometric boundary conditions:

$$(\lambda_n L) = 1.875, 4.694, 7.854, 10.995..,\ n = 1, 2,.... \tag{19}$$

The natural frequency of an elastic beam specimen was given from eq. (20),

$$(\omega_n)_b = \frac{(\lambda_n L)^2}{L^2}\sqrt{\frac{EI}{\rho A}} \tag{20}$$

where ρ is the material density and I is the moment of inertia of cross section A. The Young's modulus (E) is a function of the n[th] natural frequency of the bending mode $(f_n)_b$:

$$E = \frac{(2\pi)^2 L^4}{(\lambda_n L)^4} \frac{\rho A}{I} (f_n)_b{}^2 \quad (21)$$

The partial differential equation under free torsional vibration of an elastic beam could be obtained as follows:

$$\rho J \frac{\partial^2 \theta}{\partial t^2} = G J \frac{\partial^2 \theta}{\partial x^2} \quad (22)$$

where $\theta = \theta(x,t) = \Theta(x)\cos(\omega t + \phi)$ was the angle of torsion oscillation of the infinitesimal volume about the rotational axis and J was a polar moment inertia. The general solution of eq. (22) for the deformation mode may be written in the following form:

$$\Theta(x) = d_1 \sin\frac{\omega x}{c} + d_2 \cos\frac{\omega x}{c} \quad (23)$$

In the case of a beam with one fixed end while the other end was free, the two boundary conditions are given by:

$$\Theta(0) = 0, \quad \left.\frac{d\Theta}{dx}\right|_{x=0} = 0 \quad (24)$$

Substituting the two boundary conditions into the solution of eq. (22), the frequency equation was given by:

$$\cos\left(\frac{\omega L}{c}\right) = 0 \quad (25)$$

Where $$c = \sqrt{G/\rho} \quad (26)$$

From eq. (24), the natural frequency and eigen-functions of torsion vibration were given by:

$$(\omega_m)_t = \frac{1}{L}\left(m - \frac{1}{2}\right)\pi\sqrt{\frac{G}{\rho}}, \quad \Theta_m = d_{1m} \sin\frac{\omega_m x}{c} \quad (27)$$

The shear modulus (G) is a function of the m^{th} natural frequency of torsion mode $(f_m)_t$:

$$G = \frac{(2\pi)^2 L^2}{\left(m - \frac{1}{2}\right)^2 \pi^2} \rho (f_n)_t^2, \quad m = 1, 2, \ldots \quad (28)$$

where $\left(m - \frac{1}{2}\right)^2 \pi^2 = \frac{12 c b^2}{a^2 + b^2}$ for $m = 1$

can be represented as the first torsion mode and c is given analytically as a function of the ratio a/b, as shown in Table 1.

[Table 1] Coefficients for torsion of the rectangular specimen

a/b	c
1	0.328
1.3	0.454
1.5	0.473
2	0.486
3	0.553
5	0.801

For an effective homogeneous material, the Poisson's ratio ν could be calculated by:

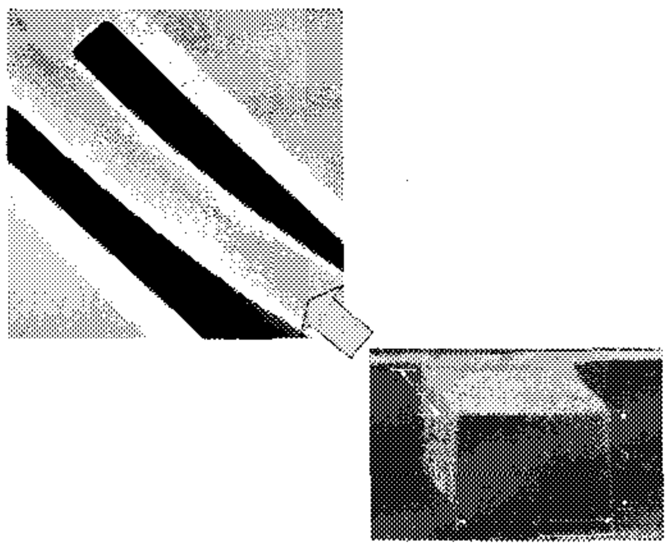
$$\nu = \frac{E}{2G} - 1 \quad (29)$$

3. Experiments and simulations

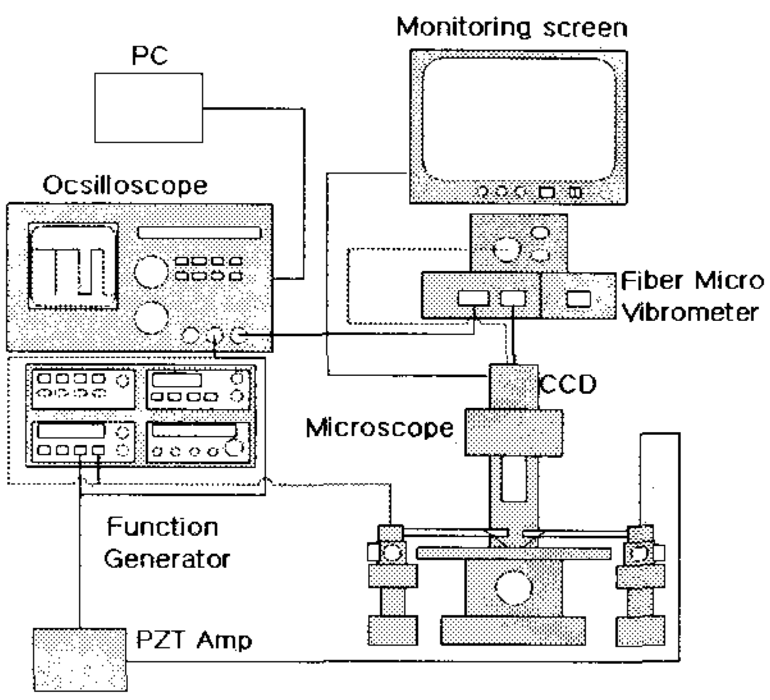
Fabricated test specimens were excited with a PZT actuator as the shaker for the frequency response measurement. The input signal for the shaker driving was generated by a function generator. The response of the cantilever was measured by a micro laser vibrometer (Polytec).

The vibrometer laser beam was focused on the end point of the cantilever structure. A microscope showed the focus point of the laser and the feature of the cantilever during the experiments. The signal data measured was transferred from the micro laser vibrometer to the oscilloscope. A PC restored the data from the oscilloscope and analyzed the data in the frequency domain. The n-th order resonant frequency was measured by the experimental system shown in figure 1. Figure 2 shows the schematic configurations of the experiment apparatus.

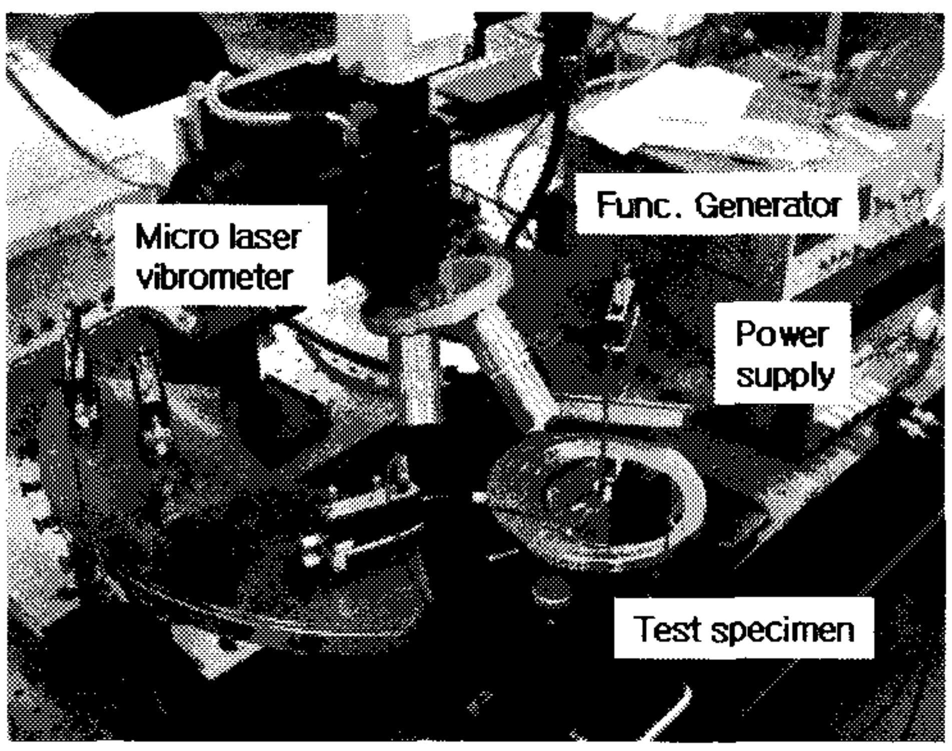
Microstructures undergo a large response at their natural frequency, which are coincident with the driving frequency of the shaker. The typical responses of the bending and torsion modes of a specimen through a base excitation are shown in table 2. The geometric dimensions of the specimens were: 200 ㎛ wide, 150 ㎛ thick and 1000 ㎛ long these dimensions are a, b and L in figure 1, respectively. Ten specimens were used to investigate the distributions of the material properties.



[Figure 1] Geometric dimensions of the polysilicon specimen [1]



(a) Schematic configurations



(b) Experimental set-up

[Figure 2] Experiment apparatus

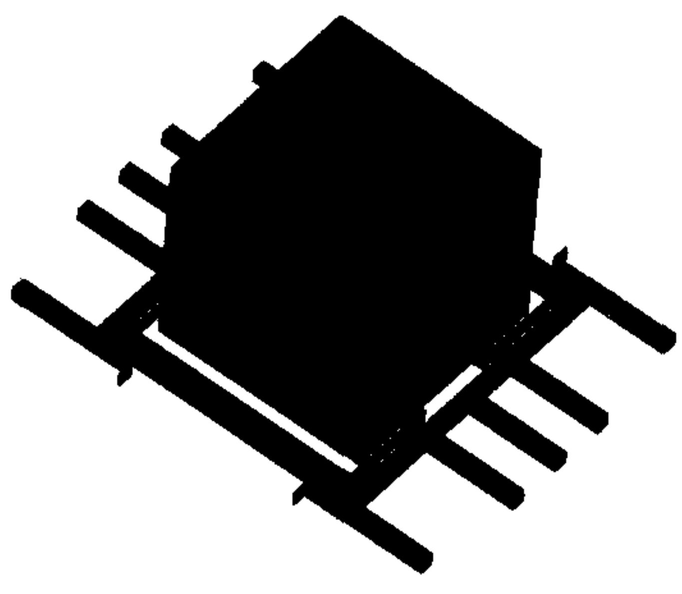
[Table 2] Typical frequency and mode of a specimen

No. of mode	Frequency (Hz)	Remark
1	1.828662E+05 $\pm\alpha_1$	1st vertical bending
2	2.406009E+05 $\pm\alpha_2$	1st lateral bending
3	1.048260E+06 $\pm\alpha_3$	2nd vertical bending
4	1.068838E+06 $\pm\alpha_4$	1st torsion
5	1.304516E+06 $\pm\alpha_5$	2nd lateral bending

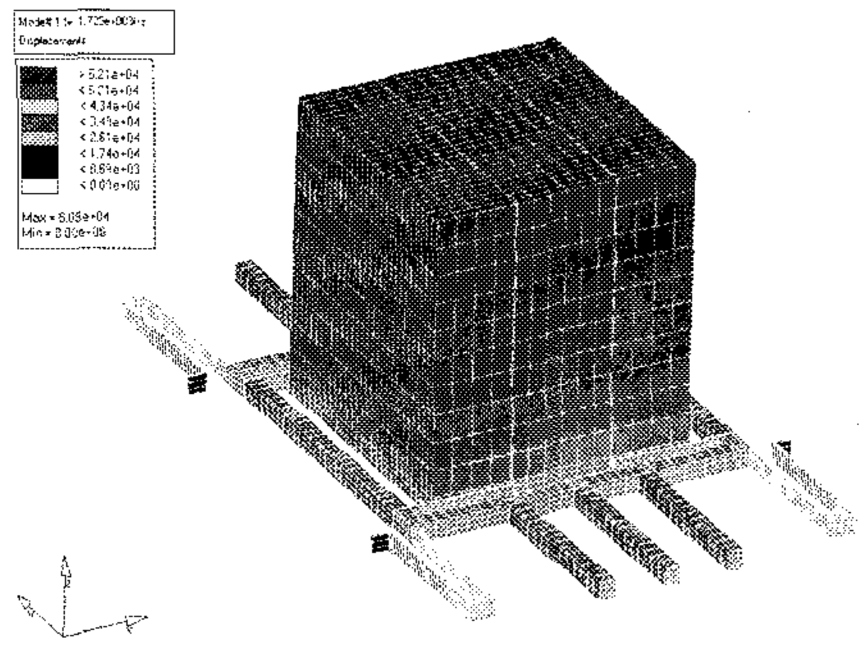
After completing the natural frequency distribution measurements for the micro- fabricated cantilever, the material properties could be calculated from eqs. (21), (28) and (29). The distributional material properties of several specimens made from polysilicon are represented in Table 3. Using the distributions of the material properties, the required natural frequency was simulated by a MEMS gyro, as shown in figure 3. Table 4 shows the comparison of its' first and second modes with the test results. The condition of simulation shows as follows: node points(5838), number of element(10060), using by Hexa elements, body fixed condition, E=179GPa, Poisson's ratio(0.27), Rho=2.33e-6kg/mm^3.

[Table 3] Distributions of material properties

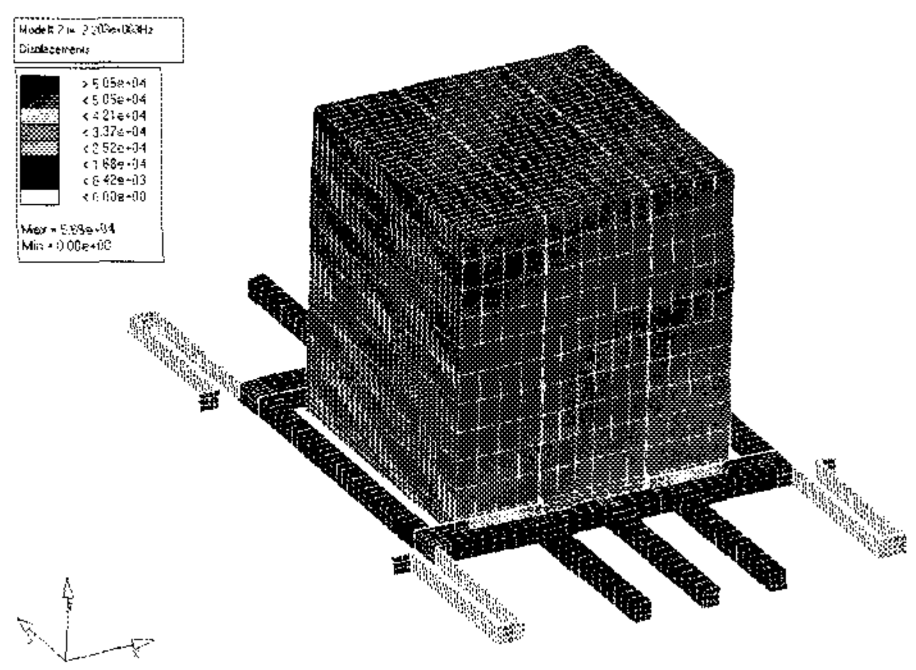
Material properties	Value
Young's modulus (GPa)	148.4±8.6
Shear modulus (GPa)	58.59±3.35
Poisson's ratio	0.263±0.01
Density (x 10^{-9} ton/mm^3)	2.572±0.15



(a) Schematic model



(b) First mode



(c) Second mode

[Figure 3] MEMS gyro model

[Table 4] Variation of dynamic characteristics of the MEMS gyro

Modes	Frequencies (Test)	Frequencies (Simulation)
First	1715.6 Hz	1723±10.2
Second	2195.7 Hz	2203±9.4

From table 4, the measured frequency ranges of the MEMS gyro can be reliably predicted, relative to the porosity-measuring method [1]. The natural frequency of value order shows about 1 of 100 between table2 and table 4. The order of table 2 shows considerable large order than table 4 because of testing specimen of simply supported beam shape. But, real model as table 4 shows considerable small order because of large mass.

4. Conclusions

This paper presented alternatives to predict the distributions of material properties through use of numerical equations and natural frequency determined from a dynamic test these included the Young's modulus, shear modulus, and Poisson's ratio of microstructure. Through the proposed technique, the reliable design of a mechanical structure is possible and the stochastic distribution of the material properties with a microstructure can be known. The result of quantitatively comparison between the distributions of material properties through use of numerical equations and dynamic test is as follows. Natural frequency of numerical analysis is a little higher than dynamic test because one of numerical analysis is neglected some properties such as porosity of microstructure.

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Jeong-Ick Lee [Regular Member]

- 1991. 2 : Dept. of Mechanical Eng. College of Engineering, Hanyang Univ. (Bachelor of Eng.)
- 1993. 2 : Dept. of Precision Mechanical Eng. College of Engineering, Hanyang Univ. (Master of Eng.)
- 1999. 8 : Dept. of Precision Mechanical Eng. College of Engineering, Hanyang Univ. (Ph.D.)
- 1993-1999 : Daewoo Electronics Company, Research Center (Senior Researcher)
- 2000-2006 : Yong-In Songdam College, Dept. of Automotive Mechanical Design (Professor)
- 2007-now : Dept. of Mechanical Design, Inha Techinical College (Professor)

<Concerning Field>
CAD/CAM/CAE, Factory Automation, Manufacturing Automation, Mold Injection, Ubiquitous Eng., MEMS, Biomechanics

Moon-Su Yeo [Regular Member]

- 1980. 2 : Dept. of Mechanical Eng. College of Engineering, Hanyany Univ. (Bachelor of Eng.)
- 1982 2 : Dept. of Mechanical Eng. College of Engineering, Hanyang Univ. (Master of Eng.)
- 1992. 8 : Dept. of Mechanical Eng. College of Engineering, Hanyang Univ. (Ph.D.)
- 1982-1997 : Agency for Defense Development (Senior Researcher)
- 1997.3 - now : Dept. of Automotive Eng., Inha Techinical College (Professor)

<Concerning Field>
CAD/CAM/CAE, Hydraulic & Pneumatic Control, Machine Automation, Automotive Engine, Vibration Control