

## Development of a Simple Rate-Sensitive Model II (Material Parameters and Modification)

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### 간단한 전단속도 의존적 모델의 개발 II (모델변수 및 간략화)

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**Abstract** This study presents the analysis of the identifications and determinations of the material parameters in the developed model in the former paper and their effects on the stress paths. It was shown that the influences of the parameters, specially involved in the strain rate and the viscous nucleus, were in generally acceptable range. From this point, the model was modified by identifying the plastic yield surface and the viscous yield surface in the same mathematical form. The modified model was successful in simulating stress path.

**Key Words** : rate-sensitive, model, elastic, plastic, viscous, parameter

**요약** 본 논문에서는 전편에 개발된 간단한 전단속도 의존적 구성모델의 각 모델변수에 대한, 정의, 값 결정법 및 결과에 미치는 영향 등에 대하여 분석하였다. 분석결과, 특히 전단속도 및 점성 핵 관련 모델변수의 영향이 보편적 범위 내에 있는 것으로 나타났다. 이러한 관점에서 점성 및 소성 항복면을 동일시하는 개념의 모델 간략화 수정을 하였으며, 결과는 비교적 성공적으로 나타났다.

## 1. Introduction

The evaluation of a constitutive model is performed generally in the basis of the three principles(1.soundness of theoretical background, 2.easiness in computer implementation, 3.material parameter). The material parameter indicates that the parameters should be clearly identified, and their values should be determined using the conventional testing methods: furthermore, their effects on the simulation results should not be so sensitive. This is very important in practical usage of a constitutive model. A rate-sensitive model was developed, in the former paper, by incorporating Adachi's rate-dependent relation into the mathematical and conceptual frame of the elastoplastic-generalized viscous theory [1][2][3][4][5][6].

In this study, details on the material parameters of the

developed model were described then the model was further modified based on the investigation of the sensitivity of each material parameter.

## 2. Material Parameters

### 2.1 Identifications and Determinations

Table 1 presents the identifications of the material parameters required for the developed model. No. 1 to No. 3 are the traditional critical state parameters. CSL and NCL respectively denote the critical state line and normally consolidation line. Their values can be obtained from the triaxial test results conducted to the critical or ultimated condition. It needs to be noted that the space or the coordinate should be correctly confirmed when the

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parameter values are determined.

[Table 1] Material Parameters

No.	Parameter	Identification
1	$M$	slope of CSL
2	$\lambda$	slope of NCL
3	$\kappa$	slope of swelling line
4	$c$	anisotropic hardening parameter
5	$x$	
6	$s_v$	viscous nucleus parameter
7	$\hat{V}$	overstress function parameter
8	$n$	overstress parameter
9	$m'$	rate-dependent parameter.

No. 4 and No. 5 are the anisotropic hardening parameters used to simulate the plastic behavior of the specimen in terms of the anisotropic modified Cam-clay model. The parameters are evaluated from the results of the anisotropic triaxial test[5]. No. 6 through No. 8 are the parameters involved in both the Perzyna's generalized viscous theory and the Adachi's rate-sensitive relation through the overstress function  $\Phi$  in Eq. (1), which serves in the way of the loading index in classical plasticity[1][2][3][4]. The viscous parameter values are obtained from the triaxial test.

$$\Phi = \frac{1}{\hat{V}} \exp \left( \frac{\frac{1}{2} \left[ \frac{N \times \frac{1}{3} I_1}{m'} \ln \frac{\epsilon_{11}^{(1)}}{\epsilon_{11}^{(2)}} - \sqrt{2} J_2^{(1)} \right]^2}{N I_1}} \right) \times (\Delta \hat{\sigma})^n \quad (1)$$

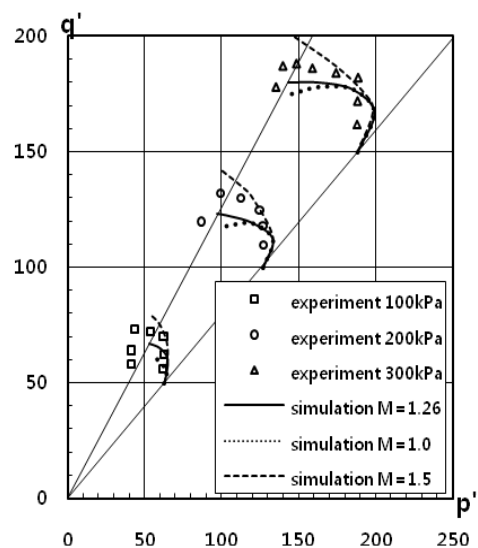
$s_v$  gives the overstress value through a closed form solution.  $m'$  is found solely in the Adachi's rate-dependent equation and its value is obtained from the triaxial test. Details on Eq. (1) were described in the former paper.

### 2.2 Sensitivities and Discussions

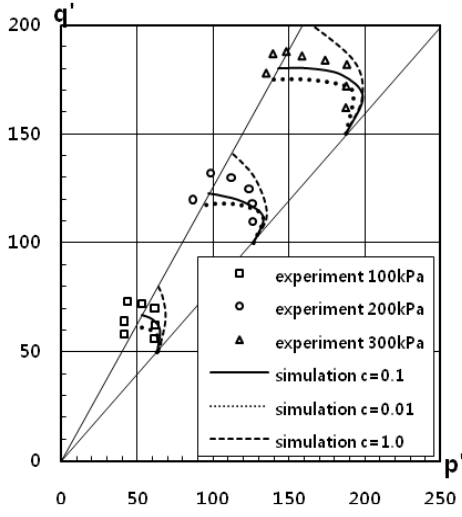
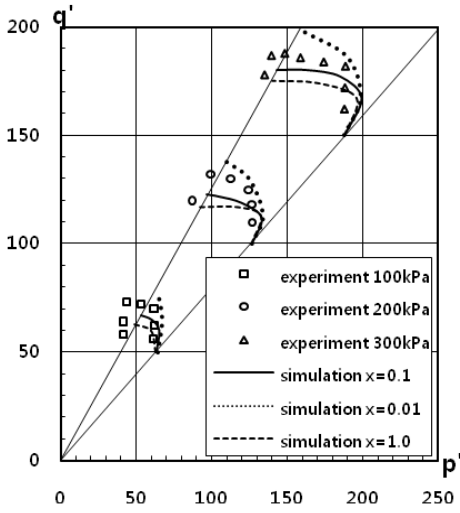
Figure 1 shows the sensitivities of the plastic material parameters on stress paths. The comparisons were made at the rate of 0.005%/min, at which the experimental data were obtained, for more reliable investigations.

The  $M$  affected, as expected, the stress path near the end rather than at the initial part. It is relatively easy to determine the value of  $M$  as the last experimental points in the stress-strain curves of the triaxial tests performed at several confining stresses have been regarded with conviction as the representation of the critical state. The  $M$  was evaluated 1.26 in this study and which was used commonly for the all simulations, but nevertheless the simulations presented the reasonable results. The  $M$  is one of the most important parameters in the critical state soil mechanics with the  $\kappa$  and  $\lambda$ . The  $M$ , that means the final part of the stress-strain behavior, is not influenced by the environment (confining stress, etc.) but it is affected by the soil's inherent nature. It is in the critical state soil mechanics that the course of the stress-strain behavior can be reasonably predicted once the last part, the critical phase, is known. Accordingly, the  $M$  is considered most essential in all models based on the critical state theory.

The  $c$  and  $x$  are the anisotropic hardening parameters. They do not seem so influential and sensitive in the stress path, as shown in Figure 1; however, it needs to be noted that the simulation of the anisotropic nature of soils became possible by just adding the parameters to the isotropic modified Cam-clay model. The differentiation of the inherent and induced anisotropy needs further research.



(a) Sensitivity of  $M$


 (b) Sensitivity of  $c$ 

 (c) Sensitivity of  $x$ 

[Fig 1] Effects of Varying Plastic Parameters on Stress Paths (0.005%/min)

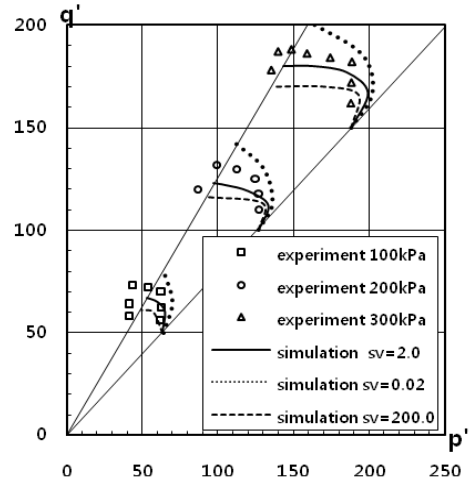
Figure 2 presents the sensitivities of the viscous and rate-sensitive parameters on stress paths. The role of  $s_v$  is in the determination of the overstress, which is used for the overstress function. The  $s_v$  does not look so sensitive. This denotes that it might be a kind of good parameter, on the other hand, it could get down to a scalar or even dropped out of the constitutive equation. Actually, the  $s_v$  was not used in the modified developed model described in the later section, yet the  $s_v$  might be expected to get more influential and sensitive at the higher rates. The  $\hat{V}$

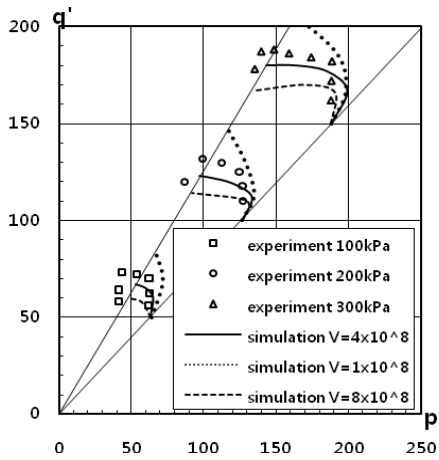
seems to be sensitive much since it was considered as a constant instead of a combined equation with several parameters. The  $\hat{V}$  have been thought of as either a constant or a combined equation as in eq. (2) with other parameters[7].

$$\hat{V} = V \frac{1}{1 + \left\langle \left( \frac{2}{3} e_{ij}^i e_{ij}^i \right)^{0.5} - \epsilon_m \right\rangle} \quad (2)$$

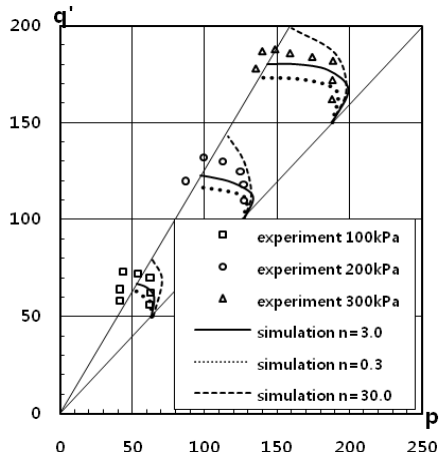
where  $e_{ij}^i$  is the inelastic deviatoric strain tensor and  $\left( \frac{2}{3} e_{ij}^i e_{ij}^i \right)^{0.5}$  is the accumulated inelastic deviatoric strain tensor.  $V$  and  $\epsilon_m$  are the model parameters related to the viscous behavior. The  $\hat{V}$  was considered as a constant in this study to avoid getting lost in the robust equation and to draw to clearer parametric study.

The  $n$  is expected to have an great effect on stress path considering its mathematical form but its influence was less great than expected for the overstress was not so large. The  $m'$  seems to affect the stress path as much as the  $n$ . The sensitivities of the parameters  $n$  and  $m'$  need to be investigated at higher rates as they are typically rate-sensitive parameters. Several mathematical forms have been proposed for the parameters based on the Roscoe's energy theory, from which the parameters were originally derived[8]. The  $n$  and  $m'$  were considered as constants since the developed model focuses on the practical use.

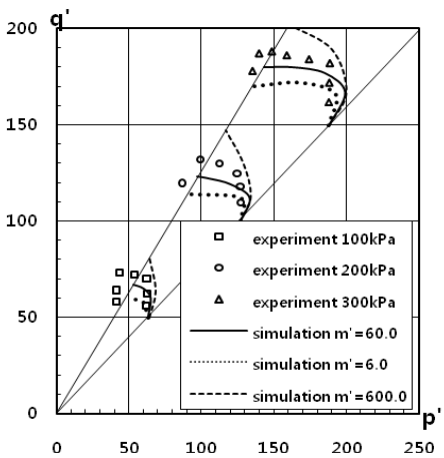

 (a) Sensitivity of  $s_v$



(b) Sensitivity of  $\hat{V}$



(c) Sensitivity of  $n$

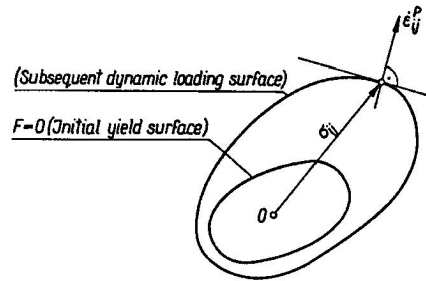


(d) Sensitivity of  $m'$

[Fig 2] Effects of Varying Viscous and Rate-sensitive Parameters on Stress Paths (0.005%/min)

### 3. Modification of the Model

The viscous theory was modified to reduce the number of the material parameters for simpler and practical usage of the model. In Perzyna's generalized viscous theory, there are two surfaces (initial yield surface and dynamic loading surface) and a stress point exists just on the dynamic yield surface as shown in Figure 3, which is different from in the classical plasticity.



[Fig 3] Generalized Viscous Theory[4]

The generalized viscous theory was simplified as: the initial yield surface and the dynamic loading surface are not differentiated so only one loading surface exists to separate only elastic deformation at the stress state inside the surface from both elastic and viscous deformation at the stress state on the surface. It was further assumed that the loading surface has the exactly same functional form and hardening rules with the plastic loading function.

In accordance with the conceptual and mathematical simplification stated above, the overstress  $\Delta \hat{\sigma}$  and the relevant parameters  $s_v$  and  $n$  disappear. Consequently, the overstress function changes eq. (1) to eq. (3).

$$\Phi = \frac{1}{\hat{V}} \exp \left( \frac{\left( \frac{1}{2} \left[ \frac{N \times \frac{1}{3} I_1}{m'} \ln \frac{\epsilon_{11}^{(1)}}{\epsilon_{11}^{(2)}} - \sqrt{2 J_2^{(1)}} \right]^2 \right)}{N I_1} \right) \times \dot{\sigma}_{ij} \quad (3)$$

The details on the simulation results performed using the simplified model were not presented on account of space considerations. The results were not so good compared with those from the model not simplified yet but gave a series of results to a reasonable and hopeful extent. It can be thought that the disappeared parameters

were not so influential considering the characteristics of the specimen.

## 4. Conclusions

Details on the material parameters of the constitutive model developed in the prior paper in series were presented in this study. The sensitivities of the material parameters were not so generally large as expected. Further simplification of the model conducted in the sense of reducing the material parameters gave the successful results.

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<Research Interests>

Soil&Foundation, Soft Soils, Ground Exploration&Testing, Numerical Analysis, Constitutive Relations