

# Automatic Mesh Generation System for FE Analysis of 3D Crack

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## 3차원 균열의 유한요소해석을 위한 자동요소분할 시스템

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**Abstract** This paper describes an automatic mesh generation system for finite element analysis of three-dimensional cracks. It is consisting of fuzzy knowledge processing, bubble meshing and solid geometry modeler. This novel mesh generation process consists of three sub-processes: (a) definition of geometric model, i.e. analysis model, (b) generation of bubbles, and (c) generation of elements. One of commercial solid modelers is employed for three-dimensional crack structures. Bubble is generated if its distance from existing bubble points is similar to the bubble spacing function at the point. The bubble spacing function is well controlled by the fuzzy knowledge processing. The Delaunay method is introduced as a basic tool for element generation. Practical performances of the present system are demonstrated through several mesh generations for 3D cracks.

**요약** 본 논문은 3차원 균열의 유한요소해석을 위한 자동요소분할시스템에 대한 것으로, 논문의 구성은 퍼지지식 처리, 버블메싱, 솔리드모델러로 구성된다. 새로운 요소분할과정은 (a) 해석모델인 기하학적 모델 정의, (b) 버블생성, (c), 요소분할로 구성되어 진다. 3차원 균열체를 위해 범용솔리드 모델러를 사용하였으며 버블은 존재하는 버블점들간의 거리가 그 점에서의 버블간격 함수와 유사한지를 결정하여 발생되어 진다. 버블간격 함수는 퍼지지식처리에 의해 잘 조절되어 진다. 요소생성에 관한 기본 툴로서는 데라우니기법이 사용되었다. 시스템의 실제적인 효용성을 검증하기 위해 3차원 균열에 대한 몇가지 예를 나타내었다.

**Key Words** : Finite Element Method, Mesh Generation, Bubble Method, Delaunay Triangulation Method

## 1. Introduction

Three dimensional cracks such as surface or embedded cracks are more common flaws being found in practical structures. Analyses of the 3D cracks are desirable in structural integrity studies of practical structures. Also, loads for pre- processing and post-processing are increasing rapidly in accordance with an increase of scale and complexity of analysis models to be solved. Particularly, the mesh generation process, which influences computational accuracy as well as efficiency and whose fully automation is very difficult in three-dimensional cases, has become the most critical

issue in a whole process of the finite element (FE) analyses. In this respect, various researches[1-4] have been performed on the development of automatic mesh generation techniques. Also, many researchers have endeavored to improve the performance of the finite element method.

Among them, the finite element method (FEM) seems the most promising method to deal with 3D crack problems because of its flexibility and extensibility. However, there are still some problems to be solved. The main concern for the FEM is a relatively higher computation cost, especially when dealing with 3D crack problems. To overcome this, several techniques such as

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the direct method[2,5], the virtual crack extension method[6], the superposition method[7] and the special singular element method [8,9] have been proposed in conjunction with the FEM. It should be also noted here that the data preparation for 3D crack analyses require special element arrangement near the crack front, and that much efforts are necessary to generate such special meshes. However in reality, labour intensive tasks to prepare a FE model of a structural component with 3D cracks are still a bottle neck.

In the present study, by integrating bubble mesh generator and one of commercial solid modeler, a new fuzzy-based system for analyzing the 3D cracks was developed.

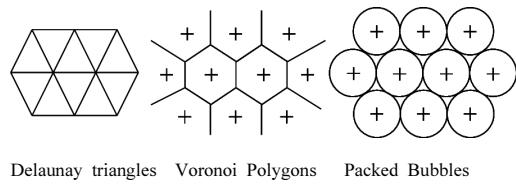
## 2. Bubble Packing Method

Bubble meshing is summarized as a sequence of two steps: (a) packing circles or spheres, called bubbles, closely in a domain, and (b) connecting their centers by Delaunay triangulation, which selects the best topological connection for a set of nodes by avoiding small angles.

### 2.1 Bubble packing

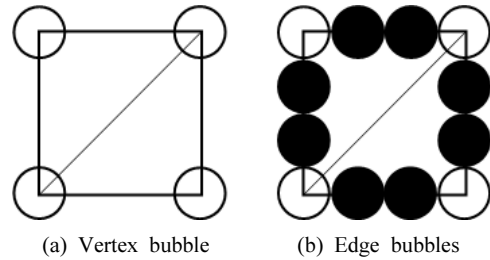
The key element of bubble meshing lies in the first step, that is, the optimization of mesh node locations by close packing bubbles. In this method, bubbles move in a domain until forces between bubbles are stabilized, and the Delaunay triangulation is then applied to generate a mesh connecting the nodes defined by the bubble packing. A repulsive or attractive force much like an intermolecular van der Waals force is assumed to exist between two adjacent bubbles. A globally stable configuration of tightly packed bubbles is determined by solving the equation of motion.

Fig. 1 shows the Delaunay triangulation and the bubble packing method, and Fig. 2 shows the procedure of the bubble packing method. Bubble meshing generates a two-dimensional triangular mesh by the following two steps: (a) Solving the equation of motion on vertices, edges, and faces(or loops) in that order, (b) Generation of triangular mesh by connecting the center points of bubbles by Delaunay Triangulation.



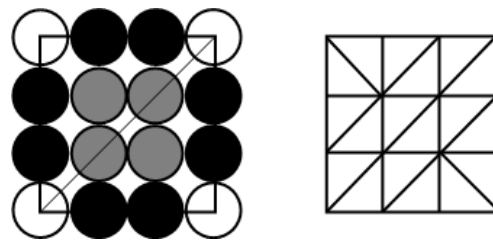
Delaunay triangles Voronoi Polygons Packed Bubbles

[Fig. 1] Schematics of the Delaunay triangulation, the Voronoi Polygons, and packed bubbles.



(a) Vertex bubble

(b) Edge bubbles



(c) Loop bubbles

(d) Final mesh

[Fig. 2] Procedure of the bubble mesh

Similar steps are also applied to the generation of three-dimensional tetrahedral meshes. In this procedure, the mesh density is needed to determine the radius of bubble. To handle general bubble spacing, we adopted a function of bubble density distribution. In the present system each bubble data are stored as a tree structure of domain such as vertices, edges, surfaces. In general, it is not so easy to well control element size for a complex geometry. A bubble density distribution over a whole geometry model can be constructed. A user selects some of local bubble patterns, depending on their analysis purposes, and designates where to locate them.

analysis system which require such special mesh, an automatic bubble mesh generation combined with the automatic mesh generation system.

### 2.2 Dynamic bubbles

A force function  $f(r)$  between two adjacent bubbles is

shown in Fig. 3. If  $f(r)=r_0$ , where  $r_0$  is defined to be

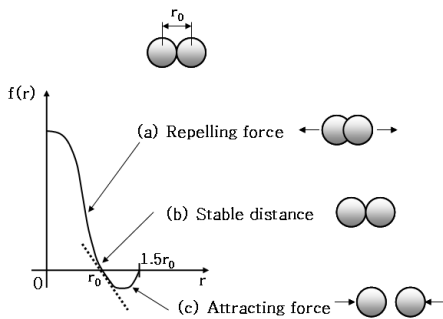
$$r_0 = 0.5(d_i + d_j) \tag{1}$$

where  $d_i, d_j$  denote diameters of two adjacent bubbles.

The two bubbles are defined to be in a stable distance. If  $f(r)$  is larger than zero, a repulsive force is assumed to exist between two bubbles, and if  $f(r)$  is smaller than zero, an attractive force is assumed to exist between them. The kinetic equation is written as follows:

$$m_i \frac{d^2 \mathbf{s}_i}{dt^2} + c \frac{d \mathbf{s}_i}{dt} = \mathbf{f}_i(t), \quad i = 1, 2, \dots, n \tag{2}$$

where  $m_i$  denotes the mass of bubble,  $c$  the coefficient of viscosity,  $\mathbf{s}_i$  the position of the  $i$ -th bubble and  $f_i(t)$  the force between two adjacent bubbles. The system of equation (2) describes the process of physical relaxation, which eventually moves the bubbles to proper equilibrium positions. The force  $f_i(t)$ , which depends on the position  $\mathbf{s}_i$



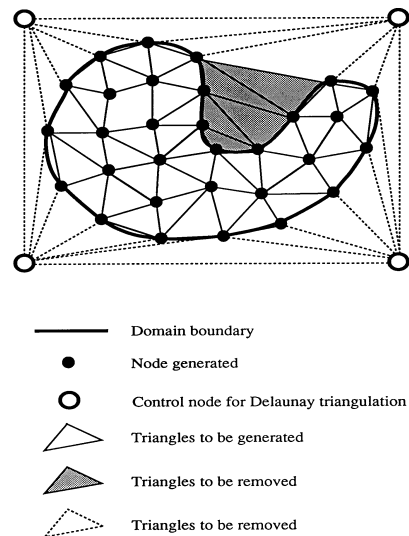
[Fig. 3] Interbubble proximity-based forces

and the distances from its center to the centers of the neighboring bubbles, is modeled by the van der Waals force.

By solving above equation by Runge-Kutta method, the optimized bubble configuration is obtained. In the process of the optimization, population of bubbles is adaptively controlled using fuzzy knowledge process. That is, excess bubbles which significantly overlap their neighbors are removed, and new bubbles are added around open bubbles which lack the appropriate number of neighboring bubbles. After the optimization of the bubble configuration, a triangular mesh is generated by using Delaunay triangulation.

### 3. Mesh Generation

The Delaunay triangulation method[1,3] is utilized to generate tetrahedral elements from numerous bubbles given in a geometry. The speed of element generation by the Delaunay triangulation method is proportional to the number of nodes. If this method is utilized to generate elements in a geometry with indented shape, elements are inevitably generated even outside the geometry as shown in Fig. 4. However, such mis-match elements can be removed by performing the IN / OUT check for gravity center points of such elements. In addition, it is necessary to avoid the generation of those mis-match elements crossing domain boundary by setting node densities on edges to be slightly higher than those inside the domain near the boundaries.



[Fig. 4] Techniques of avoiding mis-match elements

### 4. Fuzzy Knowledge Processing

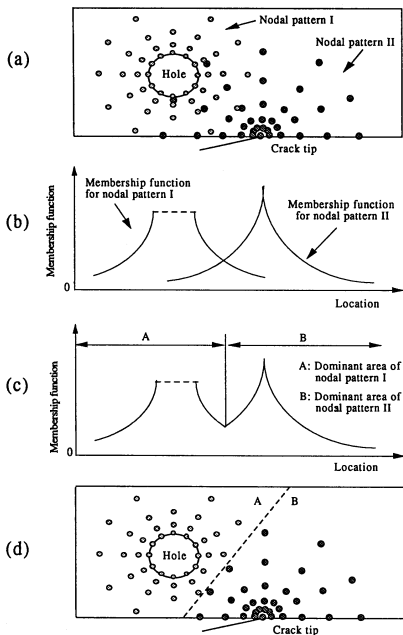
In the present method, the field A close to the crack-tip and the field B close to the hole are defined in terms of the membership functions used in the fuzzy set theory as shown in Fig. 5(c).

For the purpose of simplicity, each membership function is given a function of one-dimension in the figure. In practice the membership function can be

expressed as  $\mu(x, y)$  in this particular example, and in 3D cases it is a function of 3D coordinates, i.e.  $\mu(x, y, z)$ .

In Fig. 5(c), the horizontal axis denotes the location, while the vertical axis does the value of membership function, which indicates the magnitude of "closeness" of the location to each stress concentration field. That is, a nodal location closer to the stress concentration field takes a larger value of the membership function. As for Fig. 5(b), choosing the mesh pattern with a larger value of the membership function in each location, one can obtain an overlapped curve of both membership functions, and the domain can be automatically divided into the following two sub-domains A and B as shown in Fig. 5(c) : the sub-domain close to the crack-tip and that of the hole. Finally, both node patterns are smoothly connected as shown in Fig. 5(d). This procedure of bubble generation, i.e. the connection procedure of both bubble patterns, is summarized as follows :

- If  $\mu_A(x_p, y_p) \geq \mu_B(x_p, y_p)$  for a node p ( $x_p, y_p$ ) belonging to the pattern A, then the node p is generated, and otherwise p is not generated.
- If  $\mu_B(x_q, y_q) \geq \mu_A(x_q, y_q)$  for a node q ( $x_q, y_q$ ) belonging to the pattern B, then the node q is generated, and otherwise q is not generated.



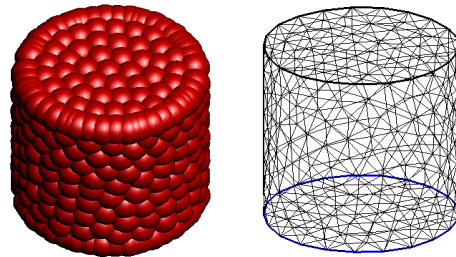
[Fig. 5] Superposition of node(bubble) patterns based on knowledge processing

It is apparent that the above algorithm can be easily extended to 3D problems and any number of node patterns. In addition, since finer node patterns are generally required to place near stress concentration sources, it is convenient to let the membership function correspond to node density as well.

## 5. Examples and Discussions

The performance of the system is demonstrated through the mesh generation of several geometries. Fig. 6 shows the uniform bubble and mesh.

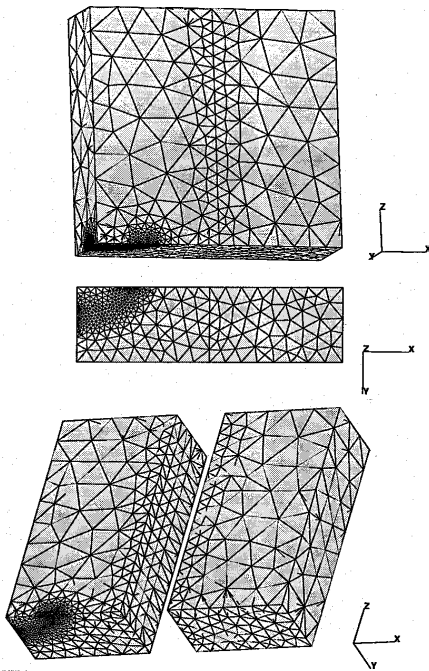
In order to examine efficiency and accuracy of the present system, a semi-elliptical surface crack in a inhomogeneous plate subjected to uniform tension was selected.



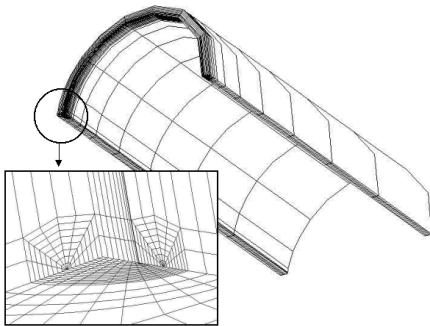
[Fig. 6] Bubble image and mesh of cylinder

Fig. 7 shows a typical finite element mesh of a quarter portion of a plate with a semi-elliptical surface crack generated by the present system. The mesh consists of 6,482 quadratic tetrahedral elements and 10,294 nodes. Nodes and elements are generated in about 90 seconds and in about 45 seconds, respectively. To complete this mesh, the following three node patterns are utilized : (a) the base node pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special node pattern for the semi-elliptical surface crack, and (c) a special node pattern in which node density is getting coarser departing from the bottom face including the surface crack and the ligament section.

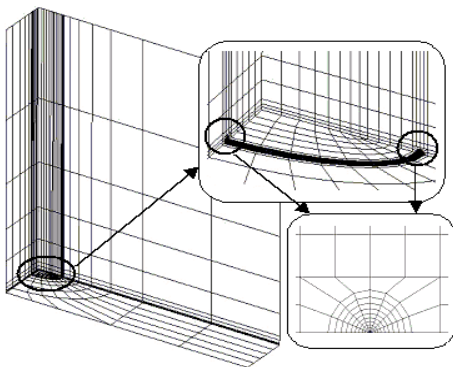
Figs. 8~9 show the examples of the application of this mesh generator for three-dimensional geometry. As shown in figures, a uniform mesh and a nonuniform mesh were connected very smoothly.



[Fig. 7] A typical mesh of a quarter portion



[Fig. 8] Examples of mesh generation for cylindrical structures.



[Fig. 9] Examples of locally-optimum mesh generation for a crack.

## 6. Conclusions

A new automated mesh generation system for finite element analysis of 3D cracks was developed. The automatic finite element mesh generation using bubble packing based on the fuzzy theory was integrated in the system, together with one of commercial solid modelers. The key features of the present algorithm are an easy control of complex 3D bubble density distribution with a fewer input data by means of the fuzzy knowledge processing technique. The effective of the present system is demonstrated through several mesh generations for 3D structures.

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<Research Interests>

Structural Analysis, Integrity Evaluation

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