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Communication Equalizer Algorithms with Decision Feedback based on Error Probability

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오류 확률에 근거한 결정 궤환 방식의 통신 등화 알고리듬

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Abstract For intersymbol interference (ISI) compensation from communication channels with multi-path fading and impulsive noise, a decision feedback equalizer algorithm that minimizes Euclidean distance of error probability is proposed. The Euclidean distance of error probability is defined as the quadratic distance between the probability error signal and Dirac-delta function. By minimizing the distance with respect to equalizer weight based on decision feedback structures, the proposed decision feedback algorithm has shown to have significant effect of residual ISI cancellation on severe multipath channels as well as robustness against impulsive noise.

요 약 이 논문에서는, 통신 채널의 다중경로에 의한 심볼간 간섭 (ISI)과 와 임펄스 잡음을 극복하도록 하기 위해 오류 확률의 유클리드 거리를 최소화하는 결정 궤환 등화 알고리듬을 제안하였다. 오류 확률의 유클리드 거리는 오류 확률과 델타 함수의 차이로 정의하였다. 등화기 가중치에 대해 유클리드 거리를 최소화함으로써 제안한 알고리듬은 임펄스 잡음에 대한 강건성 뿐 아니라 심각한 다중경로 채널의 잔여 ISI를 제거하는 효과를 보였다.

Key Words : Error probability, Decision feedback, Impulsive noise, Gaussian kernel, Multipath

1. Introduction

Most communication channels are contaminated by Gaussian noise and also impulsive noise [1,2]. Impulsive noise occurs in communication systems like power line, digital subscriber line systems, mobile radio systems [3][4] and many types of satellite communication link [5]. Measurements of the levels of impulsive noise relevant to satellite-mobile radio systems have been reported for rural, suburban, urban environments and roads carrying high density, fast moving traffic [6]. Methods for counteracting multi-path fading and impulsive noise effects are in great demand and channel equalization techniques have been used for that purpose [7].

Among various adaptive equalizer algorithms, the least mean square (LMS) algorithm [7] employing the minimum squared error (MSE) criterion has been being widely used due to its efficiency. But the LMS algorithm has a drawback that its performance is highly dependent on instant error power and affected by large error values from impulsive noise.

Instead of MSE criterion that utilizes error power, the information-theoretic learning (ITL) method has been introduced based on a combination of a nonparametric probability density function (PDF) estimator for error samples and a procedure to compute information potential [9]. As a robust ITL-type algorithm, the Euclidean distance minimization between PDFs has been introduced by Jeong et al. and applied successfully to the classification problem with a real biomedical data set

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[10]. The researchers in [10] proposed to reuse the previously acquired training–phase output samples in the test phase so that the test-phase output PDF follows the training-phase output PDF. In the research [11] for blind equalization, the Euclidean distance minimization method is applied using signal power for blind equalization. In the process of developing its unsupervised algorithm, a supervised approach that minimizes Euclidean distance of error probability (MEDE) has been introduced for communication channels with additive white Gaussian noise (AWGN).

In this paper, we investigate the performance of supervised linear MEDE algorithm for channels distorted by multipath fading and impulsive noise. Also we propose a MEDE algorithm with decision feedback (MEDE-DF) for enhanced performance against strong impulsive noise and multi-path fading.

2. Supervised MSE Criterion and Linear LMS Algorithm

The tapped delay line (TDL) with L taps (weights) is usually used as a linear structure for equalization. In that structure, input vector $\boldsymbol{X}_k = [x_k, x_{k-1}, x_{k-2}, ..., x_{k-L+1}]^T$ and the weight vector $\boldsymbol{W}_k$ at symbol time k produce an output sample $y_k = \boldsymbol{W}_k^T \boldsymbol{X}_k$. If we define that d_k is a training symbol at time k or the desired value, the error is calculated as $e_k = d_k - y_k$. The most widely used criterion, MSE is statistical average of error power $MSE = E[e_k^2]$. Instead of estimating the expected value of error power, we can use the instant squared error (SE) as a cost function.

$$SE = e_k^2 \tag{1}$$

In order to minimize the cost function (1), we apply the gradient $\frac{\partial e_k^2}{\partial \boldsymbol{W}}$ to the steepest descent method, and we obtain the well known LMS algorithm [8] with μ_{LMS} as a convergence parameter.

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + 2\mu_{LMS} e_k \boldsymbol{X}_k \tag{2}$$

We see that large error values coming from impulsive noise can affect significantly the weight adjustment process of LMS algorithm.

3. Supervised Euclidean Distance Criterion for Error Probability and Linear MEDE Algorithm

In this section we introduce briefly a supervised MEDE algorithm designed to create a concentration of error samples near zero using Euclidean distance minimization. For MEDE algorithm, the Euclidean distance between the two PDFs, the error signal PDF $f_E(e)$ and Dirac-delta function $\delta(e)$ is constructed and minimized. Defining the distance as $ED[f_E(e), \delta(e)]$ and minimizing it, we have a sharp impulse shape located at the origin of the PDF of system error [11].

$$ED[f_E(e), \delta(e)] = \int [f_E(\xi) d\xi - \int \delta(\xi)]^2 d\xi \tag{3}$$

The term $\int \delta^2(\xi) d\xi$ inside (3) can be treated as a constant c since it is not related with the weights of the adaptive system. So we have

$$ED[f_E(e), \delta(e)] = \int f_E^2(\xi) d\xi + c - 2 f_E(0) \tag{4}$$

Using the Parzen estimator [8] with Gaussian kernel and a block of N past error samples, the error PDF $f_E(e)$ can be calculated non-parametrically as

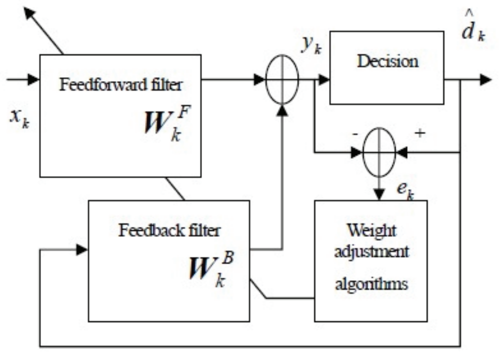
$$\begin{aligned} f_E(e) &= \frac{1}{N} \sum_{i=k-N+1}^{k} G_\sigma(e - e_i) \\ &= \frac{1}{N} \sum_{i=k-N+1}^{k} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-(e-e_i)^2}{2\sigma^2}] \end{aligned} \tag{5}$$

Then the gradient becomes

$$\frac{\partial ED[f_E(e), \delta(e)]}{\partial \boldsymbol{W}} = -\frac{1}{2\sigma^2 N^2} \sum_{i=k-N+1}^{k} \sum_{j=k-N+1}^{k} (e_j - e_i)$$

$$\cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\boldsymbol{X}_j - \boldsymbol{X}_i] + \frac{2}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot\frac{\partial y_i}{\partial \boldsymbol{W}} \qquad (6)$$

Now we adopt the gradient descent method for the minimization of cost function (4) with respect



[Fig. 1] Decision feedback equalizer.

to weight, and we have MEDE algorithm for supervised linear equalization [11] as

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k - \frac{\mu_{MEDE}}{\sigma^2 N}\left[\frac{1}{2N}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\boldsymbol{X}_j - \boldsymbol{X}_i] - 2\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot \boldsymbol{X}_i\right] \qquad (7)$$

4. Supervised MSE criterion and linear LMS Algorithm

In order for the MEDE algorithm (7) to be stretched and applied to structures with DF that consist of a feed-forward filter with weight vector $\boldsymbol{W}_k^F$ and a feedback filter with weight vector $\boldsymbol{W}_k^B$, the algorithm has to be augmented with DF part using produced decisions $\hat{d}_k$. While the feed-forward filter receives input x_k to produce output y_k, the feed back filter receives the sequence of decisions as depicted in Fig.1.

The feedback filter plays a role of removing the residual ISI from the present estimate which is caused by previously detected symbols [12]. It is noticeable that incorrect decisions can cause error propagation because the decisions are fed back into feedback filter. Though errors from AWGN do not have disastrous effects on the performance, strong impulsive noise induces substantially large error propagation. Conventional algorithms highly dependent on instant error power can not cope with this problem. Therefore equalizers with DF for impulsive noise environments have the need for robust algorithms against strong impulsive noise.

Let the number of weights in feed-forward and feedback filter section be A and B, respectively. Then output y_k of the TDL equalizer with decision feedback becomes

$$y_k = \sum_{a=0}^{A-1} w_{k,a}^F x_{k-a} + \sum_{b=0}^{B-1} w_{k,b}^B \hat{d}_{k-b-1} \qquad (8)$$

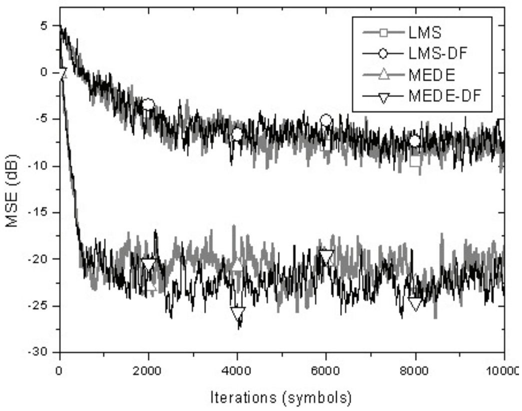
where $\{w_{k,0}^F, w_{k,1}^F, w_{k,2}^F, ..., w_{k,A-1}^F\}$ are elements of feed-forward weight vector $\boldsymbol{W}_k^F$, $\{w_{k,0}^B, w_{k,1}^B, w_{k,2}^B, ..., w_{k,B-1}^B\}$ are elements of feedback weight vector $\boldsymbol{W}_k^B$. The elements of vector $\hat{\boldsymbol{D}}_{k-1}$, $\{\hat{d}_{k-1}, \hat{d}_{k-2}, ..., \hat{d}_{k-B-2}\}$ are previously detected symbols. Now the filter weights are adjusted recursively to minimize the cost function (4) using the calculated error $e_k = d_k - y_k$ (in training mode). Then the feed-forward weight vector $\boldsymbol{W}_k^F$ and the feedback weight vector $\boldsymbol{W}_k^B$ are updated based on steepest descent method as the following.

$$\boldsymbol{W}_{k+1}^F = \boldsymbol{W}_k^F + \mu_{MEDE-DF}\frac{\partial ED[f_E(e), \delta(e)]}{\partial \boldsymbol{W}^F} \qquad (9)$$

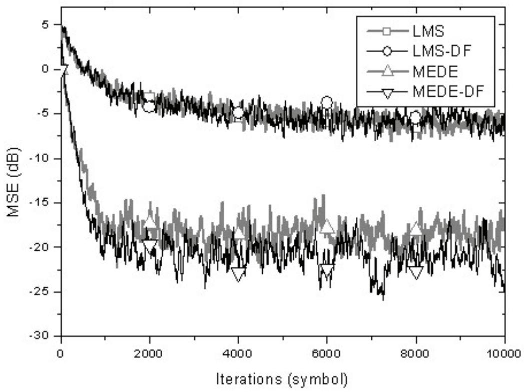
$$\boldsymbol{W}_{k+1}^B = \boldsymbol{W}_k^B + \mu_{MEDE-DF}\frac{\partial ED[f_E(e), \delta(e)]}{\partial \boldsymbol{W}^B} \qquad (10)$$

where the gradients are

$$\frac{\partial ED[f_E(e), \delta(e)]}{\partial \boldsymbol{W}^F} = \frac{1}{\sigma^2 N}\left[\frac{1}{2N}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[\boldsymbol{X}_j - \boldsymbol{X}_i] - 2\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot \boldsymbol{X}_i\right] \qquad (11)$$



[Fig. 2] MSE convergence performance for $H_1(z)$.



[Fig. 3] MSE convergence performance for $H_2(z)$.

$$\frac{\partial ED[f_E(e),\delta(e)]}{\partial \boldsymbol{W}^B} = \frac{1}{\sigma^2 N}\left[\frac{1}{2N}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j-e_i) \cdot G_{\sigma\sqrt{2}}(e_j-e_i)[\hat{\boldsymbol{D}}_{j-1}-\hat{\boldsymbol{D}}_{i-1}] - 2\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot \hat{\boldsymbol{D}}_{i-1}\right] \tag{12}$$

Now MEDE algorithm with DF (MEDE-DF) in the expression of weight elements can be rewritten as

$$w_{k+1,a}^F = w_{k,a}^F - \frac{\mu_{MEDE-DF}}{\sigma^2 N}\left[\frac{1}{2N}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j-e_i) \cdot G_{\sigma\sqrt{2}}(e_j-e_i)[x_{j-a}-x_{i-a}] - 2\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot x_{i-a}\right] \tag{13}$$

$$w_{k+1,b}^B = w_{k,b}^B - \frac{\mu_{MEDE-DF}}{\sigma^2 N}\left[\frac{1}{2N}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j-e_i) \cdot G_{\sigma\sqrt{2}}(e_j-e_i)[\hat{d}_{j-1-b}-\hat{d}_{i-1-b}] - 2\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot \hat{d}_{i-1-b}\right] \tag{14}$$

It is noticeable that the arguments of Gaussian kernels in MEDE and MEDE-DF algorithms have the effect of cutting out large error values which are mainly induced from impulsive noise.

5. Results and Discussion

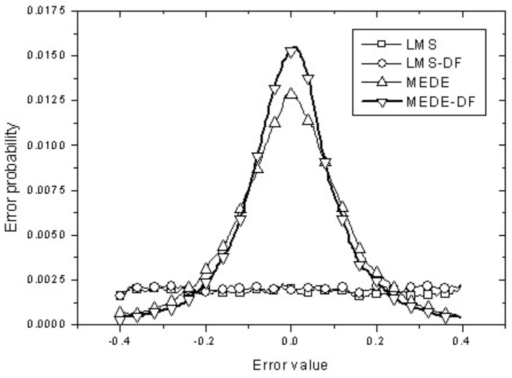
In this section we compare MSE convergence and steady state error probability of the proposed MEDE-DF algorithm, linear MEDE, LMS and LMS-DF in the multipath channel environments with impulsive noise. The multipath channel models [12] have the following transfer functions, where $H_2(z)$ is severer channel model with eigenvalue spread ratio 21 than $H_1(z)$ that has 11.

$$H_1(z) = 0.26 + 0.93z^{-1} + 0.26z^{-2} \tag{15}$$

$$H_2(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \tag{16}$$

The zero-mean white impulsive noise which is added to the channel output has the following probability density [13,14]. The value σ_2^2 is the variance of impulse noise plus back ground noise and σ_1 represents the standard deviation of back ground noise. In this simulation $\varepsilon =$ 0.03, $\sigma_1^2 = 0.001$, and $\sigma_2^2 = 50.001$.

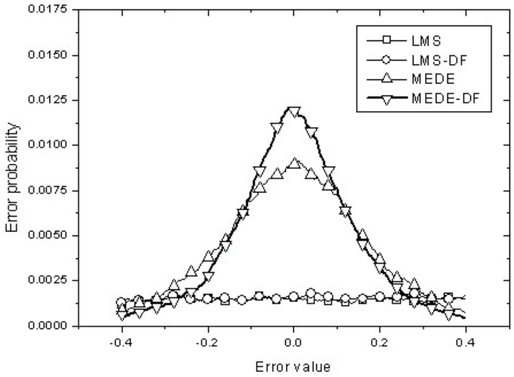
$$f_{NOISE}(n) = \frac{\varepsilon}{\sigma_2\sqrt{2\pi}}\exp[\frac{-n^2}{2\sigma_2^2}] + \frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}}\exp[\frac{-n^2}{2\sigma_1^2}] \tag{17}$$



[Fig. 4] Probability density for errors in $H_1(z)$.

The numbers of feed-forward and feedback filter weights are $A = 7$ and $B = 4$, respectively. The linear algorithms have corresponding number of weights 11. The 4PAM random symbols {-3,-1,1,3} are transmitted to the channel. The step-sizes are set to $\mu_{MEDE} = \mu_{MEDE-DF}$ =0.04, and $\mu_{LMS} = \mu_{LMS-DF}$ = 0.002 for both channel models. Data - block size N for MEDE and MEDE-DF is 20 and the kernel size σ is 0.7. The parameters are chosen to show the lowest steady-state MSE. As a figure of merit, MSE convergence, probability densities for errors are compared.

The MSE performance in Fig. 2 shows that in impulsive noise environments steady state MSE of LMS algorithm does not decrease below -6 dB regardless of decision feedback. On the other hand the MEDE converges rapidly to about -21 dB and MEDE-DF to -23 dB of steady state MSE. The performance in channel model $H_2(z)$ with impulsive noise, shows clear improvement coming from decision feedback in Fig.3. As expected, LMS and LMS-DF show the same degraded steady state MSE performance. On the other hand, the steady-state MSE performance of MEDE and MEDE-DF reach around − 18 dB and -21 dB respectively. The 3 dB of performance enhancement in $H_2(z)$ has been made by employing DF in MEDE algorithm. Performance difference is observed from the error PDF estimates in Fig. 4 and the MEDE-DF algorithm produces error distribution being the most concentrated around zero. In Fig. 5, performance differences are shown more clearly as MEDE-DF produces error distribution more concentrated around zero than MEDE while the error values of LMS, LMS-DF appear not to gather well around zero. These results indicate that DF approach to MEDE algorithm has significant effect of residual ISI cancellation on severe multipath channels with impulsive noise.



[Fig. 5] Probability density for errors in $H_2(z)$.

6. Conclusion

In this paper, a nonlinear MEDE algorithm with decision feedback is proposed to counteract multi-path fading and impulsive noise. The proposed MEDE-DF algorithm has shown the immunity to impulsive noise and the ability of the feedback filter section to cancel the remaining ISI as well. Also MEDE-DF has shown performance enhancement of above 3 dB of steady state MSE compared with linear MEDE. From the simulation results, we may conclude that MEDE algorithm has a superior resistance to impulsive noise compared to MSE-based LMS algorithm and the proposed DF approach to MEDE algorithm has significant effect of residual ISI cancellation on severe multipath channels as well as robustness against impulsive noise.

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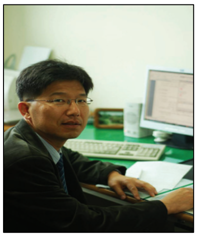

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