

Blind Algorithms using a Random-Symbol Set under Biased Impulsive Noise

Namyong Kim^{1*}

¹School of Electronic, Info. & Comm. Engineering, Kangwon National University.

바이어스 된 충격성 잡음 하에서 랜덤 심볼 열을 이용한 블라인드 알고리즘

김남용^{1*}

¹강원대학교 전자정보통신공학부

Abstract Distribution-matching type algorithms based on a set of symbols generated in random order provide a limited performance under biased impulsive noise since the performance criterion for the algorithms has no variables for biased signal. For the immunity against biased impulsive noise, we propose, in this paper, a modified performance criterion and derived related blind algorithms based on augmented filter structures and the distribution-matching method using a set of random symbols. From the simulation results, the proposed algorithm based on the proposed criterion yielded superior convergence performance undisturbed by the strong biased impulsive noise

요 약 랜덤 순서로 발생된 심볼 열을 기반으로 하는 확률 분포 매칭 타입의 알고리즘들은 그 성능 기준이 바이어스된 신호에 대한 변수를 지니고 있지 않아서 바이어스된 충격성 잡음 하에서는 제한된 성능을 나타낸다. 이 논문에서는 바이어스된 충격성 잡음을 이겨내기 위한 수정된 성능기준을 제안하고, 증강된 필터구조와 랜덤 심볼 열을 사용하는 확률 분포 매칭 방법에 기초한 블라인드 알고리즘을 제안하였다. 시뮬레이션 결과로부터 제안된 성능기준에 의해 만들어진 제안된 알고리즘이 바이어스된 강한 충격성 잡음에 대해 동요됨이 없이 탁월한 수렴성능을 보였다.

Key Words : Blind, Equalizer, Biased impulsive noise, Distribution-matching, Random symbol set

1. Introduction

Multipath fading and additive noise distort communication channels so that they induce intersymbol interference (ISI) and system instability that make communication systems unreliable [1]. In underwater communications [2], indoor communications [3], optical fiber communications and in-vehicle signal transmission [4] and digital TV systems [5], biased impulsive noise occurs causing equalizer algorithms to fail to cancel ISI properly.

Recently, to deal with ISI and impulsive noise

problems, the distribution matching criteria have been introduced. One is to minimize the distance between the distribution of error signal and Dirac-delta function [6], and another is to minimize the distance between the distribution and the distribution of a set of generated symbols in random order at the receiver [7].

Given a modulation scheme, a set of random symbols can be generated at the receiver to have the same distribution as the transmitted symbol points have. Based on the random symbols, blind equalizer algorithms have been developed matching the probability density function

*Corresponding Author : Namyong Kim(Kangwon National Univ.)

Tel: +82-01-7188-5872 email: namyong@kangwon.ac.kr

Received October 16, 2012

Revised (1st February 26, 2013, 2nd March 26, 2013)

Accepted April 11, 2013

of outputs and the distribution function of the random symbols generated according to the modulation schemes. The Gaussian kernel of the algorithm is intrinsically insensitive to impulse-infected outputs and forcing to move output samples toward corresponding symbol points, the method has produced significant performance enhancement in impulsive noise environments.

However, the method provides a limited performance under biased impulsive noise since the performance criterion has no variables for biased signal. We have also observed in this work that the algorithm performed poor in that situation.

In order for the method using a set of random symbols to have the immunity against biased impulsive noise, we propose a modified performance criterion and related blind algorithms based on the method using a set of random symbols.

2. Distribution-matching Method using a Set of Random Symbols

The performance criterion based on distribution-matching approach using the source distribution $f_s(s)$ on the axis of a source variable s and the equalizer output distribution $f_r(y)$ is defined in [7] as

$$Cost = \int f_s^2(\beta)d\beta - 2 \int f_s(\beta)f_r(\beta)d\beta \quad (1)$$

The distribution-matching criterion employs kernel density estimation method to estimate distribution functions [8]. Given a set of N samples $\{s_1, s_2, \dots, s_N\}$, the kernel density estimation method using a zero-mean Gaussian kernel with standard deviation σ puts a Gaussian kernel $G_\sigma(s - s_i)$ on each of the data points s_i . Then the N Gaussian kernels are summed as in (2).

$$f_s(s) \cong \frac{1}{N} \sum_{i=1}^N G_\sigma(s - s_i) \quad (2)$$

With this kernel density estimation method (2), the distribution-matching criterion can be written as

$$Cost = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(y_j - y_i) - 2 \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(s_j - y_i) \quad (3)$$

In blind equalization, the source symbols are not known to the receiver, but if we can generate the same shape of $f_s(s)$ at the receiver without knowing the training symbols, we can utilize the cost function (2) in blind equalization. For instance, if M symbol points equally likely with a probability $1/M$ are $S_i = 2i - 1 - M$, where $i = 1, 2, \dots, M$, we can generate N/M symbols of S_1 as $\{s_1, s_2, \dots, s_{N/M}\}$, N/M symbols of S_2 as $\{s_{N/M+1}, s_{N/M+2}, \dots, s_{2N/M}\}$, and so on. Then the whole generated symbol set $\{s_1, s_2, \dots, s_N\}$ in which the generated symbols have now the same distribution as the original source distribution $f_s(s)$ is used in the Gaussian kernel $G_{\sigma\sqrt{2}}(s_j - y_i)$ of the cost function (3).

With the set of symbols $\{s_1, s_2, \dots, s_N\}$ generated in random order according to the source distribution and the system input vector $\mathbf{X}_k = [x_k, x_{k-1}, \dots, x_{k-L+1}]^T$, the weight update equation at symbol time k can be obtained as below by minimizing the cost function (3) with respect to the system weight $\mathbf{W} = [w_0, w_1, \dots, w_{L-1}]^T$.

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu \left[\frac{1}{N^2 \sigma^2} \sum_{i=k-N+1}^k \sum_{j=1}^N (s_j - y_i) \cdot G_{\sigma\sqrt{2}}(s_j - y_i) \cdot \mathbf{X}_i - \frac{1}{2N^2 \sigma^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k (y_j - y_i) \cdot G_{\sigma\sqrt{2}}(y_j - y_i) \cdot (\mathbf{X}_i - \mathbf{X}_j) \right] \quad (4)$$

For convenience, this algorithm (4) will be referred to in this paper as distribution - matching algorithm (DMA) [7].

It is noticeable that Gaussian kernel of the upper term in (4) makes the update equation insensitive to the large differences between desired symbol points and outputs caused by impulsive noise, but it does not contain any variables. This indicates that when the noise is biased and impulsive, its immunity against such noise may not be acquired. For this reason, we propose a modified cost function for blind equalization in the following section.

3. Modified Cost Function for Blind Equalization

Under biased impulsive noise, the system may produce shifted output due to the lack of compensation ability for the biased signal. In this situation it can be difficult for the adaptive system to match the output distribution to the source distribution. Upon this assumption and for the purpose of a better match between the two distributions, we propose to shift the source distribution $f_s(s)$ on the axis of a source variable s by the amount of τ as $f_s(s-\tau)$, and insert the shifted source distribution into the cost function (3) as

$$Cost = \int f_s^2(\beta-\tau)d\beta - 2 \int f_s(\beta-\tau)f_y(\beta)d\beta \quad (5)$$

With the generated symbol set $\{s_1, s_2, \dots, s_N\}$, the distribution $f_s(s-\tau)$ forced to be shifted by τ can be

$$f_s(s-\tau) \cong \frac{1}{N} \sum_{i=1}^N G_{\sigma\sqrt{2}}(s-\tau-s_i) \quad (6)$$

Then,

$$\int f_s^2(\beta-\tau)d\beta = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(y_j - y_i) \quad (7)$$

$$\int f_s(\beta-\tau)f_y(\beta)d\beta = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(s_j + \tau - y_i) \quad (8)$$

With (7) and (8), the modified cost function to be minimized becomes

$$Cost = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(y_j - y_i) - 2 \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(s_j + \tau - y_i) \quad (9)$$

4. Proposed Blind Algorithm based on the Modified Cost Function and System Augmentation

In the distribution $f_s(s-\tau)$, the generated symbol set $\{s_1, s_2, \dots, s_N\}$ is shifted in the positive direction by the amount of τ . For the convenience's sake, we define

$s_{j,biased}$ as

$$s_{j,biased} = s_j + \tau \quad (10)$$

Then the modified cost function can be expressed as

$$Cost = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(y_j - y_i) - 2 \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(s_{j,biased} - y_i) \quad (11)$$

where y_i is a function $g(\cdot)$ of weight and input as $y_i = g(\mathbf{W}^T \mathbf{X}_i)$, where the system $g(\mathbf{W}^T \mathbf{X}_i)$ could be any type of filter structures.

When we employ a tapped delay line (TDL) structure with system weights $\mathbf{W} = [w_0, w_1, \dots, w_{L-1}]^T$ and system input $\mathbf{X}_i = [x_i, x_{i-1}, \dots, x_{i-L+1}]^T$, the system output y_i at symbol time i becomes $y_i = \mathbf{W}^T \mathbf{X}_i$. If we add another weight element w_L to \mathbf{W} and a constant $-c$ to \mathbf{X}_i for system augmentation, we obtain the output of the augmented system $y_{i,aug} = \mathbf{W}_{aug}^T \mathbf{X}_{i,aug}$ with the augmented weight and input vector $\mathbf{W}_{aug} = [w_0, w_1, w_2, \dots, w_L]^T$ and $\mathbf{X}_{i,aug} = [x_i, x_{i-1}, \dots, x_{i-L+1}, -c]^T$, respectively. The augmented weight vector \mathbf{W}_{aug} has an additional weight element $w_L \cdot c \rightarrow \tau$ compared to the original weight vector \mathbf{W} . This weight vector expansion implies system augmentation. Then $y_{i,aug} = \mathbf{W}_{aug}^T \mathbf{X}_{i,aug} = \mathbf{W}^T \mathbf{X}_i - w_L \cdot c = y_i - w_L \cdot c$ and we have $y_i = y_{i,aug} + w_L \cdot c$ and $s_{j,biased} - y_i = s_j - y_{i,aug} + \tau - w_L \cdot c$.

We can observe that after inserting $s_{j,biased}$ into the modified cost function (11) and by the minimization of it, the Gaussian kernel $G_{\sigma\sqrt{2}}(s_{j,biased} - y_i)$ discards excessive argument values from impulsive noise, and the bias τ can be controllable as $w_L \cdot c \rightarrow \tau$ based on the augmented filter structure. This analysis indicates that the modified cost function on appropriate filter structures has the immunity to biased impulsive noise as well as the ability of canceling ISI.

Assuming that $w_L \cdot c = \tau$ after convergence, $s_{j,biased}$ becomes

$$s_{j,biased} - y_i = s_j - y_{i,aug} \quad (12)$$

With the relationship $y_j - y_i = y_{j,aug} - y_{i,aug}$ and (12), the gradient of the modified cost function $\partial Cost(\mathbf{W}_{aug}) / \partial \mathbf{W}_{aug}$ at the sample time k is calculated from

$$\begin{aligned} \partial Cost(\mathbf{W}_{aug}) / \partial \mathbf{W}_{aug} &= \frac{1}{N^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k \left[\frac{\partial G_{\sigma\sqrt{2}}(y_{j,aug} - y_{i,aug})}{\partial (y_{j,aug} - y_{i,aug})} \frac{\partial (y_{j,aug} - y_{i,aug})}{\partial \mathbf{W}_{aug}} \right. \\ &\quad \left. - 2 \frac{\partial G_{\sigma\sqrt{2}}(s_j - y_{i,aug})}{\partial (s_j - y_{i,aug})} \frac{\partial (s_j - y_{i,aug})}{\partial \mathbf{W}_{aug}} \right] \\ &= \frac{1}{2\sigma^2 N^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k [G_{\sigma\sqrt{2}}(y_{j,aug} - y_{i,aug})(y_{j,aug} - y_{i,aug})(\mathbf{X}_{i,aug} - \mathbf{X}_{j,aug}) \\ &\quad - 2G_{\sigma\sqrt{2}}(s_j - y_{i,aug})(s_j - y_{i,aug})\mathbf{X}_{i,aug}] \end{aligned} \quad (13)$$

Then \mathbf{W}_{aug} can be updated by the steepest descent method with (13), the random-order symbol set $\{s_1, s_2, \dots, s_N\}$, and a convergence parameter μ as

$$\mathbf{W}_{k+1,aug} = \mathbf{W}_{k,aug} - \mu \cdot \partial Cost(\mathbf{W}_{aug}) / \partial \mathbf{W}_{aug} \quad (14)$$

For convenience, the proposed algorithm summarized as (13) and (14) will be referred to in this paper as biased DMA.

5. Simulation Results and Discussion

The biased impulsive noise model in this paper is composed of the background Gaussian noise, the impulse noise and additional constant DC noise. The background noise is white Gaussian with variance σ_{GN}^2 . The impulse noise is also white Gaussian with variance σ_{IN}^2 , the average number of impulses per information symbol duration ε occurring according to a Poisson process. Then the PDF expression of the impulsive noise n_{im} (background Gaussian noise + impulse noise) is

$$\begin{aligned} f_{IM}(n_{im}) &= \frac{1-\varepsilon}{\sigma_{GN}\sqrt{2\pi}} \exp\left[-\frac{n_{im}^2}{2\sigma_{GN}^2}\right] \\ &+ \frac{\varepsilon}{\sqrt{2\pi(\sigma_{GN}^2 + \sigma_{IN}^2)}} \exp\left[-\frac{n_{im}^2}{2(\sigma_{GN}^2 + \sigma_{IN}^2)}\right] \end{aligned} \quad [7][9].$$

On the other hand, the biased impulsive noise n by the amount of bias A can have the following distribution function.

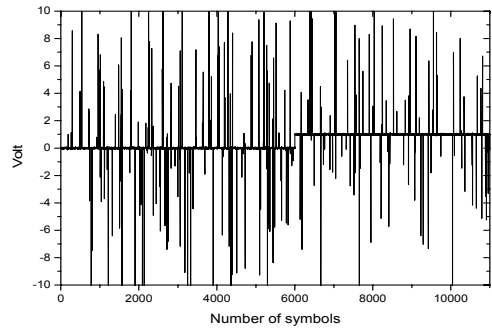
$$\begin{aligned} f(n) &= \frac{1-\varepsilon}{\sigma_{GN}\sqrt{2\pi}} \exp\left[-\frac{(n-A)^2}{2\sigma_{GN}^2}\right] \\ &+ \frac{\varepsilon}{\sqrt{2\pi(\sigma_{GN}^2 + \sigma_{IN}^2)}} \exp\left[-\frac{(n-A)^2}{2(\sigma_{GN}^2 + \sigma_{IN}^2)}\right] \end{aligned} \quad (15)$$

In this section the performance of DMA in (4) and the proposed algorithm is experimented for multipath channels contaminated with biased impulsive noise defined by (15). The transmitted symbol points are $\{\pm 3, \pm 1\}$, equi-probable. The following two channel models are adopted as in [1].

$$H_1(z) = 0.26 + 0.93z^{-1} + 0.26z^{-2} \quad (16)$$

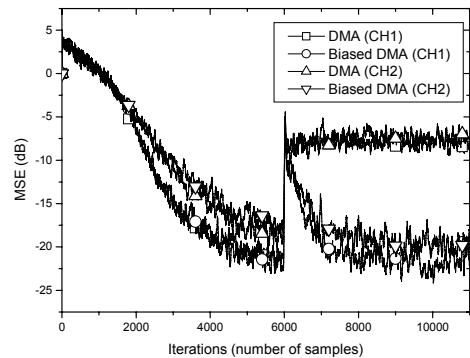
$$H_2(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \quad (17)$$

The bias $A=1$ is added at 6000 symbol time after all algorithms have converged for the assessment of the potential usefulness of the proposed algorithm. We can observe in Fig. 1 that the biased impulsive noise has excessively strong impulses frequently (some impulses reach over 20 volts) and an abrupt shift of its mean value in the mean time.



[Fig. 1] Biased impulsive noise.

MSE performance for $H_1(z)$ (CH1) and $H_2(z)$ (CH2) is shown in Fig. 2.

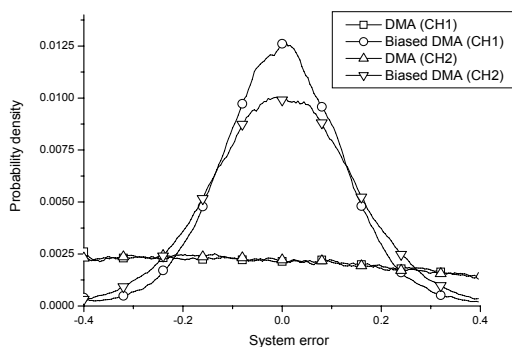


[Fig. 2] MSE performance for the channel $H_1(z)$ (CH1) and $H_2(z)$ (CH2) with biased impulsive noise.

We employ a tapped delay line structure with $L=11$ weights. The constant c in the augmented input vector $\mathbf{X}_{i,aug}$ is set to 2. Data-block size is set to be $N=20$. The kernel size σ and step-size for the algorithms are commonly 0.5 and 0.007, respectively. All these parameters are selected in the case that the algorithms have the lowest steady state MSE values.

In Fig. 2, in the transient stage, both algorithms yield the same convergence speed but from the symbol time 6000 when the biased impulsive noise is beginning to be added to the channel, DMA shows abrupt MSE increase of over 10 dB and no decrease of MSE after that. This indicates that DMA has very limited performance of ISI cancelation. On the other hand, the proposed algorithm shows the same abrupt MSE increase but instantly converges in less than 1000 symbols and stays at about -20 dB of steady state MSE. This implies that the proposed algorithm copes immediately with the biased impulsive noise. Similar results are also observed for a severer channel $H_2(z)$.

The performance can be examined more apparently in the error distribution comparison in Fig. 3. The error distributions for the proposed algorithm in both cases of $H_1(z)$ and $H_2(z)$ form a highly concentrated shape centered at zero. However, the ones of DMA are widely spread as the bias impulsive noise is added. This phenomenon can be interpreted that DMA has its well tuned weights for ISI cancellation be readjusted to cancel the bias of the biased impulsive noise, and in that process, DMA loses both functions of ISI cancelation and bias compensation.



[Fig. 3] Error distribution in the environment with biased impulsive noise.

6. Conclusion

Distribution-matching type blind algorithms based on a set of symbols generated in random order at the receiver have produced superior performance in the environments of impulsive noise. But under biased impulsive noise the methods provide inferior performance since the performance criterion for the algorithms has no variables for biased signal. In order for the distribution-matching method using a set of random-order symbols to have the immunity against biased impulsive noise, we have proposed a modified criterion that employs a shifted source distribution function and matches the output distribution by using a set of pseudo-source symbols at the receiver. Based on the proposed criterion, a new blind algorithm has been derived on an augmented filter structure. From the simulation results of blind equalization, the proposed algorithm based on the proposed criterion yielded superior performance coping with the strong biased impulsive noise. These results lead us to conclude that the proposed method can be effectively used in blind equalizers in the environments with severe ISI and strong biased impulsive noise.

References

- [1] J. Proakis, *Digital Communications*, McGraw-Hill, 2nd ed, 1989.
- [2] Z. Daifeng, Q. Tianshuang, "Underwater sources location in non-Gaussian impulsive noise environments", *Digital Signal Processing*, vol. 16, pp. 149-163, March, 2006. DOI: <http://dx.doi.org/10.1016/j.dsp.2005.04.008>
- [3] K. Blackard, T. Rappaport, and C. Bostian, "Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE J. Select. Areas Commun.*, vol. 11, pp. 991-1001, Sept. 1993. DOI: <http://dx.doi.org/10.1109/49.233212>
- [4] Y. Yabuuchi, D. Umehara, M. Morikura, T. Hisada, S. Ishiko, and S. Horiata, "Measurement and analysis of impulsive noise on in-vehicle power lines", *Proceedings of ISPLC'10*, pp. 325-330, March 2010. DOI: <http://dx.doi.org/10.1109/ISPLC.2010.5479913>
- [5] J. Armstrong, J. Shentu, C. Chai, and H. Suraweera, "Analysis of impulse noise mitigation techniques for digital television systems", *Proceedings of InOWo'03*,

- pp. 172-176, 2003.
- [6] N. Kim and Y. Hwang, "Communication equalizer algorithms with decision feedback based on error probability", *Journal of the Korea Academia-Industrial cooperation society*, vol. 12, no. 5, pp. 2930-2395, May, 2011.
DOI: <http://dx.doi.org/10.5762/KAIS.2011.12.5.2390>
- [7] N. Kim, and H. Byun, "Blind equalization based on Euclidian distance of information theoretic learning for impulsive noise environments", *Proceedings of 2010 International conference on computer communications and networks*, pp. 53-56, July 2010.
- [8] E. Parzen, "On the estimation of a probability density function and the mode," *Ann. Math. Stat.* vol. 33, p.1065, 1962.
DOI: <http://dx.doi.org/10.1214/aoms/1177704472>
- [9] I. Santamaria, P. P. Pokharel, and J. Principe, "Generalized correlation function: Definition, properties, and application to blind equalization", *IEEE Trans. Signal Processing*, vol. 54, pp. 2187-2197, June 2006.
DOI: <http://dx.doi.org/10.1109/TSP.2006.872524>

Namyong Kim

[Regular member]



- Feb. 1988 : M.S. from Yonsei Univ. in electronics
- Feb. 1991 : Ph. D from Yonsei Univ. in electronics
- Feb. 1992 ~ Feb. 1998 : An associate professor in Kwandong Univ.
- Mar. 1998 ~ current : A professor in the school of electronics, information & communications eng., Kangwon National Univ.

<Research Interests>

Adaptive Equalization, RBFN, Odour Sensing Systems