# Robust Model Predictive Speed Control for Permanent Magnetic Synchronous Motor with Linear Matrix Inequality

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> 선형행렬부등식을 활용한 영구자석형 동기모터의 강인한 모델예측속도 제어

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#### Abstract

This paper introduces a Robust Model Predictive Speed Control (RMPSC) framework for Permanent Magnet Synchronous Motors (PMSM), leveraging Linear Matrix Inequality (LMI) techniques along with a Disturbance Observer (DOB). The primary objective is to ensure stability and enhance control performance in the presence of parameter uncertainties and external disturbances affecting the PMSM. These disturbances, treated collectively as external torques, are estimated by the DOB and integrated into the RMPSC strategy. The design of both DOB and RMPSC relies on a multi-stage LMI-based optimization methodology. Moreover, the paper addresses parameter uncertainty arising from manufacturing variations or aging effects by proposing a systematic tuning approach for optimizing controller gains within defined ranges. Additionally, an offline optimization scheme is developed to minimize computational overhead, facilitating real-time implementation. Simulation results validate the efficacy and robustness of the proposed control strategy.

## 1. Introduction

The utilization of Permanent Magnet Synchronous Motors (PMSMs) is on the rise owing to their numerous advantages [1], such as heightened efficiency, compact design, precise control, elevated torque-to-inertia ratio, and superior power density. As a result, PMSMs find application across diverse fields [2], encompassing CNC machines, robotics, which traditionally demand precise control, alongside factory automation equipment, emerging renewable energy technologies, space and defense sectors, and more recently, in electric power steering (EPS) systems within the automotive industry. However, despite their widespread adoption, PMSMs pose challenges due to their time-varying nature, inherent nonlinearity, mechanical coupling with other systems, and intricate dynamics, which complicate control system design [3].

Various control techniques have been proposed to effectively manage the intricacies of PMSM systems, including modified PI control [10–14], sliding mode control (SMC) [15–18], model predictive control (MPC) [19–21], and

deep learning (DL)-based control [4]. In [4], modified control methods based on PI control were explored. For instance, a vector-controlled PMSM drive employing a hybrid fuzzy-PI speed controller was introduced, leveraging the strengths of both PI and fuzzy controllers. This approach integrates two outputs with assigned weights for predetermined speed errors or speed errors. Additionally, an innovative adaptive fuzzy proportional-integral (AFPI) method was introduced to tackle uncertain saturation, disturbances, and system delays. An online optimization scheme utilizing an adaptive fuzzy tuner was employed for AFPI control. Conversely, a mixed PI controller incorporating angle compensation, complex coefficient, and proportional integrator controllers was proposed to offer decoupling performance for digital control delay times. Moreover, a neural network-based PID controller was presented, combining four control modules for optimal gain adjustment of the online PID controller while addressing nonlinearity. However, PI control methods typically face limitations in maintaining control performance within a specific range.

This paper introduces a Robust Model Predictive Speed Control (RMPSC) framework for regulating the speed of PMSM, utilizing linear matrix inequality (LMI) techniques in conjunction with a disturbance observer (DOB). This approach aims to ensure robustness against unknown disturbances and parameter fluctuations. All external torques affecting the PMSM are classified as disturbances and estimated by the DOB. However, to enhance disturbance estimation performance, our proposed DOB formulates multiple LMIs based on the Lyapunov function. treating it as an optimization problem to derive optimal disturbance gains. Furthermore, the proposed RMPSC method, incorporating the DOB, addresses parameter uncertainties while regulating PMSM speed. We also present a gain tuning procedure to optimize observer and controller gains. This method is anticipated to ensure stable operation in environments where PMSM parameters change over time or where torque variations occur rapidly [26-27].

## 2. System description and controller design

In many papers, the mathematical model of PMSM is converted from a three-phase fixed coordinate system converted to a two-axis rotation coordinate system for convenience of analysis and control. The generalized nonlinear system dynamics of PMSM can be represented in the synchronous reference frame (SRF) as follows:

$$V_d = R_s I_d + \frac{d}{dt} \Phi_d - P \omega_m \Phi_q \tag{1}$$

$$V_q = R_s I_q + \frac{d}{dt} \Phi_q + P \omega_m \Phi_d \tag{2}$$

where  $V_d$ ,  $V_q$ ,  $I_d$ ,  $I_q$ ,  $R_s$ ,  $\Phi_d$ ,  $\Phi_q$ ,  $\omega_m$  are the stator voltage, stator current, stator resistance, stator flux linkage, and rotor speed, respectably.

The PMSM stator flux linkage is expressed as:

$$\Phi_d = L_d I_d + \lambda_r \tag{3}$$

$$\Phi_q = L_q I_q \tag{4}$$

where  $L_d$ ,  $I_q$  are the stator inductance and  $\lambda_r$  is the flux linkage of permanent magnet, respectably.

Utilizing the Euler approximation, the linear state-space model in continuous time can be converted to the following linear state-space model in discrete-time using step time. The design of a Robust Model Predictive Speed Controller (RMPSC) to regulate the speed of PMSM under parameter uncertainty and disturbance is presented in this section. At each step, the reference speed is assumed to be known and optimal control gain is determined by optimization problem of cost function. The cost function can be defined as follows based on the difference between the steady state and the steady state control input.

$$\mathbf{J}(\mathbf{k}) \coloneqq \left\| \widehat{\boldsymbol{\Psi}}(\mathbf{k}+1) \right\|_{\mathbf{H}} + \left\| \Delta \mathbf{u}(\mathbf{k}) \right\|_{\mathbf{R}}$$
(5)

Consequently, the DOB and RMPCS proposed of this paper are illustrated as shown in Figure 1



[Fig. 1] Block diagram of the proposed RMPSC method with disturbance observer

### 3. Simulation result

The simulation is performed with several cases and results are presented to demonstrate the effectiveness of the proposed RMPSC method in this section. The nominal value of PMSM parameters used in simulation are listed in Table I. As the characteristics of the surface mount PMSM used in this simulation, the stator inductance and values are the same.

Description	Value
Rated Power	0.4 kW
Stator Inductance	6.71 mH
Stator Resistance	1.55 Ohm
Flux linkage of permanent magnets	0.175 Wb
Viscous friction coefficient	0.0003 Nms
Rotational inertia	$0.0002 \ kgm^2$
Pole pairs	3
Sampling time	100 us

[Table 1] System Parameters

A simulation is carried out with parameter uncertainty added to the normal condition. The initial conditions are the same as the previous cases, the disturbance and reference speed change at each specific times, and the parameters are set to have a 20% error compared to the nominal parameters. The simulation starts from the initial conditions, and the reference speed is increased to 1800 rpm at 0.5 s., and the disturbance is changed to 3.0 Nm at 0.7 s., and the results are shown in Figure 2. The speed control results adapted with the proposed and comparative methods are shown in Figure 2(a), and the three-phase current and electromagnetic torque values and the estimated disturbance can be seen in Figure 2(b), (c), and (d) as a result of applying the proposed RMIPSC method. Figure 2(a) shows that the proposed RMIPSC method has better speed control performance compared to other comparative methods for reference speed changes and disturbances under parameter uncertainty.



[Fig. 2] Simulation results of dynamic response of PMSM when disturbance changes in PMSM with parameter uncertainty (a) Speed results of PMSM applying the PI-decoupling method, sliding mode control method, and proposed RMPSC method, respectively, (b) three-phase current of PMSM under proposed RMPSC method, (c) electromagnetic torque of PMSM under proposed RMPSC method, (d) Measured disturbance and estimated disturbance by DOB.

## 5 Conclusion

This study proposes a robust model predictive speed control (RMPSC) method for speed control of PMSM that is robust to disturbance and parameter uncertainty. Including external mechanical torque, that affect the PMSM were considered disturbances, and this value was estimated through a disturbance observer. The disturbance and state observer were designed to satisfy the Lyapunov stability and a cost function was constructed based on the steady-state of PMSM at the reference speed. Simulation results showed the effectiveness of the proposed RMPSC method in providing speed control performance of PMSM even under conditions where the target speed changes and external disturbances applied to the PMSM.

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